MPM 2D

Linear Systems Word Problem Review

June 11

Choose the most appropriate method to solve the following problems. Remember to define all variables, state your equations, show your work and give a concluding statement to answer the problem.

1. The sum of two numbers is 377 and their difference is 107. Find the numbers.

2. The sum of Lisa's age and Ellen's age is 41. Five years from now the sum of their age will be three times Lisa's present age. How old is Lisa?

3. Adult and student tickets were sold for the school play. The total receipts were $2005. The number of student tickets sold was equal to three times the number of adult tickets sold decreased by 56. If student tickets cost $3.50 and adult tickets cost $5.00, how many adult tickets were sold?

4. A bill of $424 was paid with $5 bills and $2 bills. A total of 128 bills were used. How many of them were fives?

5. A lab technician needs to make 400 kg of 51% alcohol solution by combining 60% alcohol solution by mass and 40% alcohol solution by mass. How many kilograms of the 40% solution must be used?

6. Wayne invested $9000, part at 9% per year and the remainder at 8% per year. After one year the total interest from these investments was $750. How much was invested at 9%?

7. It took Spencer 7 h to drive the 390 km from Cold Bay to Morgan's Cove. He averaged 60 km/h for the first part of the trip, but was forced to complete the trip at 50 km/h due to a thunderstorm. How many hours did he spend driving at 50 km/h?

8. Three times the larger of two numbers increased by four times the smaller is 205. Six times the larger plus 3 times the smaller is 270. Find the numbers.

9. Flying into the wind, an airliner takes 4 h to go 960 km. The same plane flying with the wind takes only 3 h to make the same trip. Find the speed of the plane and the speed of the wind in km/h.

ANSWERS:

1. 242, 135
2. 17
3. 142
4. 56 $5 bills, 72 $2 bills
5. 180 kg
6. $3000
7. 3 h
8. 31, 28
9. plane 280 km/h, wind 40 km/h

MPM2D

Analytic Geometry Review

1. Find the midpoint of the line segment between
   a) $(-3,-2)$ and $(5,6)$
   b) $(-1,5)$ and $(4,-1)$
   c) $(-2,4)$ and $(7,4)$

$\overline{M} = \frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}$

2.
   a) Draw the triangle with vertices $T(-8,6), U(2,10)$ and $V(4,-4)$.
   b) Draw the median from vertex U. Then, find the equation for this median.
   c) Draw the right bisector of TU. Then find an equation for this right bisector.

3. Find the length of the line segment between
   a) $(3,17)$ and $(8,5)$
   b) $(4,3)$ and $(-2,-5)$
   c) $(-4,-2)$ and $(5,3)$

4. Determine the length of the line segment defined by each pair of points.
   a) $T(-2,4)$ and $U(7,4)$
   b) $(V(3,-5)$ and $W(-5,-6)$

5.
   a) Determine the length of the median from vertex A of triangle ABC, $A(-5,-2)$, $B(4,-1)$, $C(1,7)$.
   b) Determine the perimeter of the triangle.

6. Show that the triangle with vertices $A(-2,1)$, $B(5,2)$, and $C(1,5)$ is isosceles.

7. A triangle has vertices $D(-2,7)$, $E(-4,2)$, and $F(6,-2)$.
   a) Show algebraically that this triangle is a right triangle.
   b) Find the midpoint of the hypotenuse.
   c) Show that this midpoint is equidistant from each of the vertices.

8. Find the shortest distance from the origin to the line defined by $y = 3x - 10$.

9. Find an equation for the circle
   a) with points $(0,4), (-4,0), (0,-4), (4,0)$.
   b) passing through the point $(-3,5)$ and has centre at the origin.
   c) has a centre at the origin and a radius of 5.4
   d) has a diameter of 14 and is centred on the origin.

10.

a) Determine whether the point $A(-2,-6)$ lies on the circle defined by $x^2 + y^2 = 40$.
b) Find an equation for the radius line from the origin to point A.
c) Find an equation for the line that passes through A and is perpendicular to OA.
d) Use a graph to check your answer for parts a), b) and c).
e) Explain why A is the only point on the line that also lies on the circle.

11.

a) Define a median. What is the difference between midpoint and median?
b) What are the steps to finding the equation of a median?
c) What are the steps to finding a right bisector equation?

12.

a) Verify that ΔDEF, $D(1,4)$, $E(-2,2)$, $F(3,1)$ is a right triangle.
b) Describe another method you could use to a answer part a).

13. Verify that the quadrilateral $JKLM, J(-3,2), K(-2,-2), L(7,1), M(3,4)$ is a trapezoid.

14. Classify the quadrilateral with vertices $T(2,4), U(8,2), V(7,-1), W(1,1)$. Justify this classification.

15. Show that $A(-12,-5)$ and $B(12,5)$ are endpoints of a diameter of the circle defined by $x^2 + y^2 = 169$.

16. Verify that points $P(5,7)$ and $Q(7,-5)$ lie on the circle with equation $x^2 + y^2 = 74$.

Answers:

1. a) (1,2) b) (1.5, 2) c) (2.5, 4)
2. b) $y = \frac{9}{4}x + \frac{11}{2}$ c) $y = -\frac{5}{2}x + \frac{1}{2}$
3. a) 13 b) 10 c) $\sqrt{106}$
4. a) 9 b) $\sqrt{65}$
5. a) $\sqrt{\frac{325}{4}}$ b) about 28.4
6. $AC = BC = 5$
7. a) $m_{DE} = \frac{5}{2}$ and $m_{EF} = -\frac{2}{5}$; therefore, $\angle DEF = 90°$
   b) (2, 2.5)
   c) The distance from the midpoint to each vertex is $\sqrt{\frac{145}{4}}$
8. $\sqrt{10}$

9. a) $x^2 + y^2 = 16$ b) $x^2 + y^2 = 34$ c) $x^2 + y^2 = 29.16$
   d) $x^2 + y^2 = 49$
10. a) point A lies on the circle. b) $y = 3x$ c) $y = -\frac{1}{3}x - \frac{20}{3}$
    e) Answers may vary. For example: On either side of point A, the circle curves away from the tangent line.
11. See your notes for the definition.
12. Two methods are:
    i. find all slopes, and two of them should be negative reciprocals
    ii. find all distances between points, and show Pythagorean theorem works.
13. Find all slopes. Two of them should be equal which means they are parallel.
14. Rectangle. Finding all slopes, there are 2 pairs of equal slopes (means parallel), there are negative reciprocal slopes (meaning right angle). Finding all distances, there are 2 pairs of equal distances, which mean equal length.
15. Answers may vary. A and B both satisfy the equation for the circle. $AB = 26$, which is exactly twice the radius of the circle.
16. Find the distances from the origin. They should equal the radius. Or show that the points satisfy the equation.

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#2. L = Lisa's age

E = Ellen's age

① $L + E = 41$ (Today)

② $(5 + L) + (5 + E) = 3L$ (5 years from today)

$10 + L + E = 3L$

$L - 3L + E = -10$

②' $-2L + E = -10$

$-$ ① $L + E = 41$

$-3L = -51$

$\frac{-3L}{-3} = \frac{-51}{-3}$

$L = 17$

sub $L = 17$ into ①

① $17 + E = 41$

$E = 41 - 17 = 4$

#3 S = # of student tickets sold → \$3.50/ticket

A = " Adult tickets sold → \$5/ticket

$3.50S + 5A = 2005$ = total rev = price x units —— ①

$S = 3A - 56$ —— ②

sub ② int ① → $3.50(3A - 56) + 5A = 2005$

$10.5A - 196 + 5A = 2005$

$15.5A = 2005 + 196$

$\frac{15.5A}{15.5} = \frac{2201}{15.5}$ ∴ $A = 142$

sub $A = 142$ into eq # (2)

$S = 3(142) - 56 = 370$

∴ They sold 370 student tickets and 142 adult tickets.

"Analytic Geometry Review"

1. c) $(-2,4)$ and $(7,4)$

$$\overline{M} = \frac{-2+7}{2}, \frac{4+4}{2}$$

$$\overline{M} = \left(\frac{5}{2}, \frac{8}{2}\right) \text{ or } \left(\frac{5}{2}, 4\right)$$

$x^2 + y^2 = r^2$

16. $P(5,7)$ $\quad Q(7,-5) \rightarrow x^2 + y^2 = 74$

$x^2 + y^2 = (\sqrt{74})^2$

$r = \sqrt{74}$

$x \quad y$

$P(5,7) \Rightarrow 5^2 + 7^2 = 74$

$25 + 49 = 74$

$74 = 74$

∴ Yes P is on the circle.

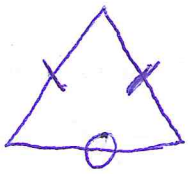
$Q(7,-5) \Rightarrow 7^2 + (-5)^2 = 74$

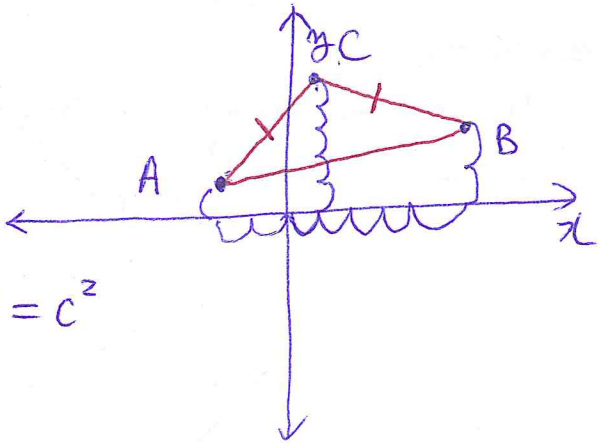
$49 + 25 = 74$

$74 = 74$

∴ Yes Q is on the circle.

#6. A $(\overset{x}{-2}, \overset{y}{1})$, B $(\overset{x}{5}, \overset{y}{2})$ and C $(\overset{x}{1}, \overset{y}{5})$ isoceles $\triangle$





$\overline{AC} = \overline{BC}$

$a^2+b^2=c^2$

$D = \sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$

$\overline{\quad} = \sqrt{(-2-1)^2+(1-5)^2}$

$\overline{AC} = \sqrt{(-3)^2+(-4)^2}$

$= \sqrt{9+16}$

$= \sqrt{25} = \sqrt{5^2} = 5$

$\overline{BC} = \sqrt{(5-1)^2+(2-5)^2}$

$\overline{BC} = \sqrt{4^2+(-3)^2}$

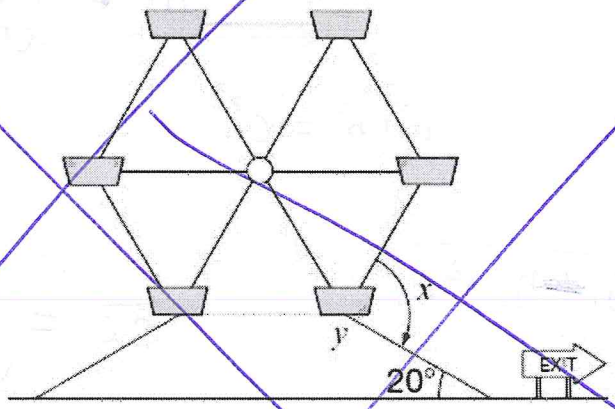
$\overline{BC} = \sqrt{16+9}$

$\overline{BC} = \sqrt{25} = 5$

$\therefore \overline{AC} = \overline{BC}$, so it is isoceles $\triangle$.

28 Wheels of Fun

A Ferris wheel has six sides of equal length. The exit ramp of the Ferris wheel is in the shape of a trapezoid and has an angle of incline of 20°.



What are the values of x and y?

Use geometric properties to justify your answer.