1. Simplify each of the following. Express each answer with positive exponents.

a) $\left(\frac{a^3}{b^{-4}}\right)^{-2}$

$= \frac{a^{-6}}{b^8}$

$= \frac{1}{a^6 b^8}$

b) $\frac{-12x^{-3}(xy^2)^{-1}}{(2x^3y^{-1})^3}$

$= \frac{-12x^{-3}x^{-1}y^{-2}}{8x^9y^{-3}}$

$= \frac{-12x^{-4}y^{-2}}{8x^9y^{-3}}$

$= -\frac{3}{2}x^{-13}y^1$

$= \frac{-3y}{2x^{13}}$

2. Evaluate without the use of a calculator. Express answers as integers or rational numbers.

a) $9^{-\frac{3}{2}}$

$= \frac{1}{9^{3/2}}$

$= \frac{1}{(\sqrt{9})^3}$

$= \frac{1}{3^3}$

$= \frac{1}{27}$

b) $64^{\frac{1}{2}} \times 27^{-\frac{1}{3}}$

$= (\sqrt{64})\frac{1}{\sqrt[3]{27}}$

$= \frac{8}{3}$

3. Solve for the unknown:

a) $27^x = \frac{1}{9}$

$(3^3)^x = \frac{1}{3^2}$

$3^{3x} = 3^{-2}$

$3x = -2$

$x = -\frac{2}{3}$

b) $2^{x+4} + 2^x = 136$

$2^x(2^4) + 2^x = 136$

$2^x(2^4 + 1) = 136$ Factor out 2^x

$2^x(16+1) = 136$

$\frac{2^x(17)}{17} = \frac{136}{17}$

$2^x = 8$

$2^x = 2^3$

$x = 3$

4. Expand and simplify $2(a+1)^2 - (2a-1)(2a+1)$

$2(a+1)(a+1) - (2a-1)(2a+1)$

$= 2(a^2+2a+1) - (4a^2-1)$

$= 2a^2 + 4a + 2 - 4a^2 + 1$

$= -2a^2 + 4a + 3$

5. Simplify and state any restrictions on the variable.

a) $\frac{a^2+13a+30}{a^2-4a} \times \frac{a-a^2}{a^2+2a-3}$

$= \frac{(a+10)(a+3)}{a(a-4)} \times \frac{a(1-a)}{(a+3)(a-1)}$

$= \frac{(a+10)(a-1)(-1)}{(a-4)(a-1)}$

$= \frac{-a-10}{a-4}$ $a \neq -3, 0, 1, 4$

b) $\frac{3x}{x-3} + \frac{2x}{x+3} = \frac{3x(x+3)}{(x+3)(x-3)} + \frac{2x(x-3)}{(x+3)(x-3)}$

$= \frac{3x(x+3)+2x(x-3)}{(x+3)(x-3)}$

$= \frac{3x^2+9x+2x^2-6}{(x+3)(x-3)}$

$= \frac{5x^2+5x}{(x+3)(x-3)}$

$= \frac{5x(x+1)}{(x+3)(x-3)}$ $x \neq -3, 3$

6. Simplify : $\frac{d^2-2d-3}{2d^2+d-1} - \frac{3d-1}{2d^2-7d+3}$

* On a separate page.

$\frac{(d-3)(d+1)}{(2d-1)(d+1)} - \frac{3d-1}{(2d-1)(d-3)}$

$= \frac{(d-3)(d+1)(d-3) - (3d-1)(d+1)}{(2d-1)(d+1)(d-3)}$

7. Simplify to mixed radical form.

a) $2\sqrt{6} \times 3\sqrt{8}$

$= 6\sqrt{48}$

$= 6\sqrt{16(3)}$

$= 6(4)\sqrt{3}$

$= 24\sqrt{3}$

b) $\frac{12\sqrt{60}}{3\sqrt{12}}$

$= 4\sqrt{5}$

c) $\frac{4\sqrt{27}-6}{6}$

$= \frac{4\sqrt{9(3)}-6}{6}$

$= \frac{4(3)\sqrt{3}-6}{6}$

$= \frac{12\sqrt{3}-6}{6}$

$= 2\sqrt{3} - 1$

8. Solve for x by factoring or using the quadratic formula: Express answer in exact simplified form.

a) $6x^2+11x-2=0$

$(6x-1)(x+2)=0$

$6x-1=0$ or $x+2=0$

$x = \frac{1}{6}$ $x=-2$

b) $3x^2 = 2x-4$

$3x^2-2x+4=0$

$b^2-4ac = (-2)^2-4(3)(4)$

$= 4-48$

$= -44$

There are no real roots.

6. continued...

$$= \frac{(d-3)(d-3)(d+1) - (3d-1)(d+1)}{(2d-1)(d+1)(d-3)}$$

$$= \frac{(d^2-6d+9)(d+1) - (3d^2+2d-1)}{(2d-1)(d+1)(d-3)}$$

$$= \frac{d^3 - 6d^2 + 9d + d^2 - 6d + 9 - 3d^2 - 2d + 1}{(2d-1)(d+1)(d-3)}$$

$$= \frac{d^3 - 8d^2 + d + 10}{(2d-1)(d+1)(d-3)}$$

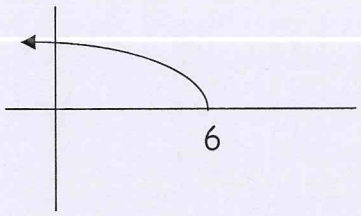
9. State the domain and range of each of the following:

a) $y = 2(x-3)^2 + 4$

$D = \{x \in \mathbb{R}\}$

$R = \{y \in \mathbb{R},\ y \geq 4\}$

b)



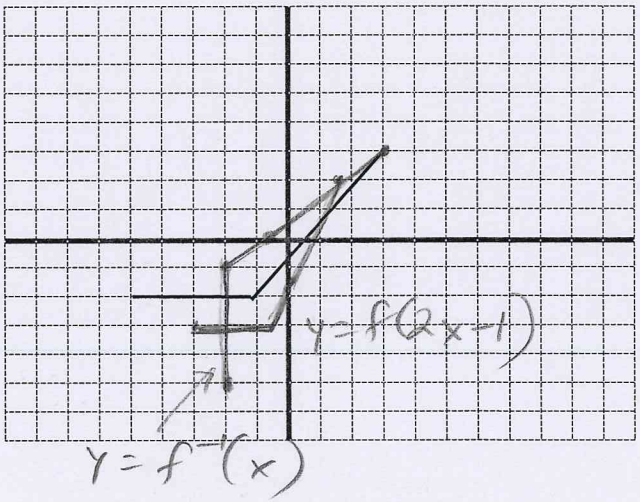
$D = \{x \in \mathbb{R},\ x \leq 6\}$

$R = \{y \in \mathbb{R},\ y \geq 0\}$

10. Given the graph below of the function f(x), sketch on the same grid (label each clearly):

a) $y = f(2x) - 1$

b) $y = f^{-1}(x)$ Flip x and y-coordinates



11. a) Determine the inverse of the function $f(x) = \sqrt{x-1}$

$y = \sqrt{x-1}$

$x = \sqrt{y-1}$

$x^2 = y - 1$

$x^2 + 1 = y$

* Careful! Domain is not just $x \in \mathbb{R}$!

$R: \{y \in \mathbb{R},\ y \geq 0\}$

↓ Domain of inverse.

$f^{-1}(x) = x^2 + 1,\ x \geq 0$

b) What is the domain and range of the inverse?

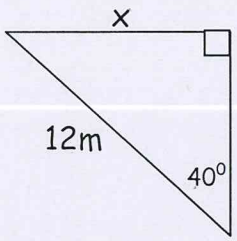
Domain of $f(x)$: $D: \{x \in \mathbb{R},\ x \geq 1\}$

∴ For inverse $D: \{x \in \mathbb{R},\ x \geq 0\}$

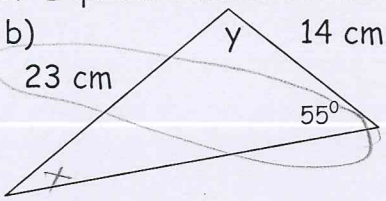
$R: \{y \in \mathbb{R},\ y \geq 1\}$

12. Solve for the unknowns in the diagrams. Express answer to the nearest tenth.:

a)



b)



matching side and angle.

$\sin 40° = \frac{x}{12}$

$12 \sin 40° = x$

$7,7 m = x$

$\frac{\sin x}{14} = \frac{\sin 55°}{23}$

$\sin x = 0,4986$

$x \approx 29,9°$

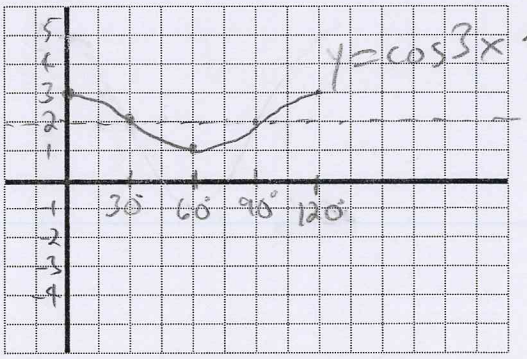
$y = 180° - 55° - 29,9° = 95,1°$

13. Sketch one cycle (or period) of $y = \cos 3x + 2$, showing all critical points.

$k = 3$

$\text{period} = \frac{360}{3}$

$= 120$



14. State the amplitude, period, phase shift and vertical shift for each of the following (where applicable):

a) $y = 2\sin(\theta - 150°)$

b) $y = -\cos\frac{1}{3}(x + 45°) + 1$

	a)	b)
amplitude	2	+1
period	360°	$\frac{360°}{(\frac{1}{3})} = 1080°$
phase shift	150° right	45° left
vertical shift	none	up 1 unit.

* amplitude is positive
negative is reflection in x-axis

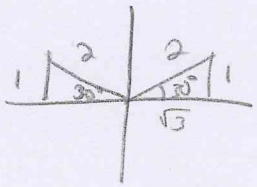
15. Find any angles that satisfy the following. Give exact answers.

a) $\sin x = 0.5,\ 0 \le x \le 360°$

S A
T C

$\sin x = \frac{1}{2}$ ← positive

∴ in Q1 and Q2



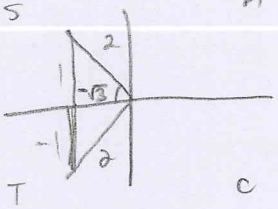
$\beta = 30°$

$x = 30°$ OK

$x = 180° - 30° = 150°$

$x = 30°, 150°$

b) $\cos x = -\frac{\sqrt{3}}{2},\ 0° \le x \le 360°$ ← Negative



$\beta = 30°$

$x = 180° - 30°$

$= 150°$

$x = 180° + 30° = 210°$

$x = 150°, 210°$

16. Find the indicated term:

a) $t_n = 1 - 2n^2;\ t_5$

$t_5 = 1 - 2(5)^2$

$= 1 - 2(25)$

$= -49$

b) $12, 19, 26, 33, \ldots\ t_{15}$ (+7 +7 +7)

$t_n = a + d(n-1)$

$t_{15} = 12 + 7(15-1)$

$= 12 + 7(14)$

$= 110$

arithmetic sequence.

17. Find the indicated sum using an appropriate formula:

a) $S_{12} = 35 + 32 + 29 + \ldots$

-3 -3

arithmetic series.

$S_{12} = \frac{12}{2}(2(35) + (-3)(12-1))$

$= 222$

b) $S_{12} = 36864 - 55296 + 82944 - 124416 + \ldots$

$\frac{-55296}{36864} = -1.5$ $\quad \frac{82944}{-55296} = -1.5$

$r = -\frac{3}{2}$

$S_{12} = \frac{36864\left(\left(-\frac{3}{2}\right)^{12} - 1\right)}{-\frac{3}{2} - 1} = -1\,898\,442$

18. Suppose that Carolyn wants to deposit enough money today in a savings account so she will have $1,000,000 in 40 years. If the account pays 7% p.a. compounded semi-annually, how much does she need to deposit?

one deposit is normal compound interest

$A = P(1+i)^n$

$i = \frac{0.07}{2} = 0.035$

$n = 40(2) = 80$

$1\,000\,000 = P(1.035)^{80}$

$\frac{1\,000\,000}{1.035^{80}} = P$

$63\,792.85 = P$

∴ She needs to deposit $63 792,85