Future value of annuity

19. If Jon deposits $100 at the end of every three months, how much will he have accumulated after 8.5 years if interest is 6% p.a. compounded quarterly?

$S_{34} = \dfrac{100\left((1.015)^{34} - 1\right)}{1.015 - 1}$

$i = \dfrac{0.06}{4} = 0.015$

$n = (8.5)(4) = 34$

$S_{34} = \$4393.31$

∴ He will have accumulated $4393.31

20. The height above the ground (in metres) of a rocket is given by the equation $h(t) = -4.9t^2 + 49t + 2.5$ where the time **t** is measured in seconds.

a) What is the height of the rocket after 2 seconds?

$h(2) = -4.9(2)^2 + 49(2) + 2.5$

$= 80.9\text{ m}$ ∴ The height is 80.9m

b) When does the rocket achieve it **maximum** height?

vertex → get it in vertex form!

$h(t) = -4.9(t^2 - 10t) + 2.5$

$= -4.9(t^2 - 10t + 25 - 25) + 2.5$

$= -4.9(t^2 - 10t + 25) + 122.5 + 2.5$

$= -4.9(t - 5)^2 + 125$

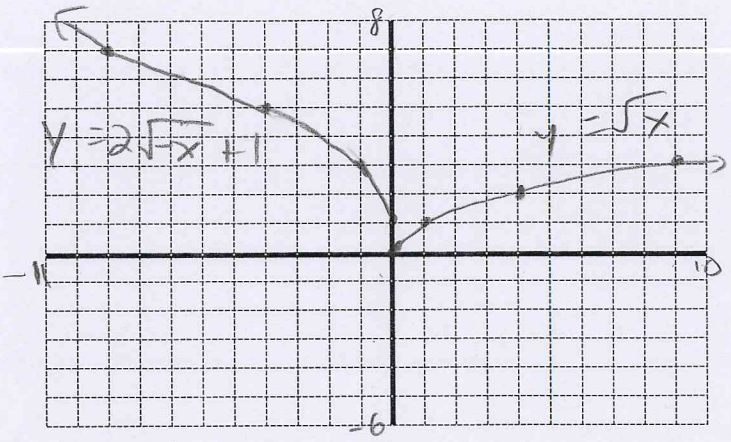
(5, 125)
t h

At $t = 5$s, the rocket reaches its max height.

c) What is its maximum height?

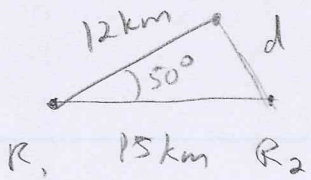
It's max height is 125m.

21. By referring to the graph of $y = \sqrt{x}$, sketch the graph of $y = 2\sqrt{-x} + 1$



22. A forest ranger saw a fire on a bearing of ~~050~~ ^{0} from her position and she estimated that the distance to the fire was approximately 12 km. Another ranger station was due east of her and approximately 15 km from her. How far from the fire was the second ranger station? (Include a diagram.)

E 50°N



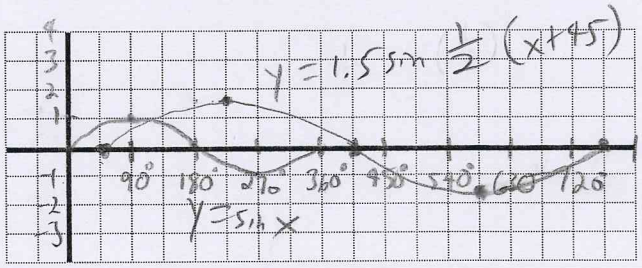
$d^2 = 12^2 + 15^2 - 2(12)(15)\cos 50°$

$= 137.6$ km

It was 137.6 km from the fire.

23. By referring to the graph of $y = \sin x$, sketch the graph of $y = 1.5\sin\frac{1}{2}(x+45)$ showing all critical points and using ~~radian~~ degree measure on your scale.

$k = \frac{1}{2}$ period $= \frac{360}{(\frac{1}{2})} = 720°$

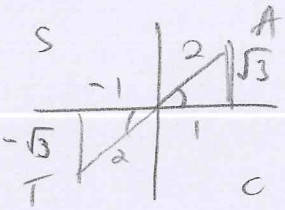


24. Solve for x and give exact answers in the indicated measure:

a) $\tan x - \sqrt{3} = 0, \quad 0° \le x \le 360°$

$\tan x = \frac{\sqrt{3}}{1}$

$\beta = 60°$



$x = 60°$ or $x = 180° + 60° = 240°$

$x = 60°, 240°$

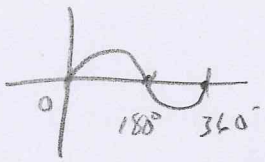
b) $2\sin x - \sin x \cos x = 0, \quad 0 \le x \le 360^0$

$\sin x (2 - \cos x) = 0$ Factor out $\sin x$

$\sin x = 0$ or $2 - \cos x = 0$

$2 = \cos x$

↑ Never bigger than 1 ; impossible



$x = 0°, 180°, 360°$

$\boxed{x = 0°, 180°, 360°}$

25. Prove the identity $\frac{2\cos^2 x}{1 - \cos^2 x} = \frac{2}{\tan^2 x}$

$LS = \frac{2\cos^2 x}{1 - \cos^2 x}$

$= \frac{2\cos^2 x}{\sin^2 x}$

$RS = \frac{2}{\tan^2 x}$

$= \frac{2}{\left(\frac{\sin^2 x}{\cos^2 x}\right)}$

$= 2 \times \frac{\cos^2 x}{\sin^2 x}$

$= \frac{2\cos^2 x}{\sin^2 x}$

$\therefore LS = RS$

26. A car's purchase price is $24,000. At the end of each year, the value of the car is three-quarters of the value at the beginning of the year.

a) Write the first four terms of the sequence of the car's value at the end of each year.

$t_0 = 24000$

a value → $t_1 = 24000\left(\frac{3}{4}\right)$

$= 18000$

$t_2 = 18000\left(\frac{3}{4}\right)^{2-1}$

$= 13500$

$t_3 = 18000\left(\frac{3}{4}\right)^{3-1}$

$= 10125$

$t_4 = 18000\left(\frac{3}{4}\right)^{4-1}$

$= 7593.75$

b) Determine the general term of this sequence to represent the value of the car after n years.

$t_n = 18000\left(\frac{3}{4}\right)^{n-1}$

c) Using your general term in b), determine when the car's value will first be less than $800?

$800 = 18000\left(\frac{3}{4}\right)^{n-1}$

$\frac{800}{18000} = \left(\frac{3}{4}\right)^{n-1}$

$0.0\overline{4} = \left(\frac{3}{4}\right)^{n-1}$

$\log 0.0\overline{4} = \log\left(\frac{3}{4}\right)^{n-1}$

$\log 0.0\overline{4} = (n-1)\log\left(\frac{3}{4}\right)$

$\frac{\log 0.0\overline{4}}{\log\left(\frac{3}{4}\right)} = n-1$

$10.82 = n-1$

$n = 11.8$

$n = 12$

After 12 years

27. Describe how the graph of $y = -2f(3x-6)+2$ can be produced from the graph of $y = f(x)$. List all transformations.

$y = -2f(3(x-2))+2$

reflection in x-axis

vertical stretch by a factor of 2

horizontal compression by a factor of $\frac{1}{3}$

shift right 2 units

shift up 2 units.

28. Suppose you win a lottery. You have two choices for receiving the money.
Choice 1: $50 000 at the end of each year for 20 years
Choice 2: $500,000 now
If current interest rates are approximately 8% p.a. compounded yearly, which is the better choice. Justify your answer with a clear solution and explanation.

Choice 1 - Future Value of annuities

$S_{20} = \frac{50000(1.08^{20}-1)}{1.08-1}$

$= 2288098.22$

Choice 2 - compound interest

$A = P(1+i)^n$

$= (500000)(1.08)^{20}$

$= 2330478.57$

choice 2 is better.

29. The element Californium "*A*" has a half life of approximately 45 minutes. How long would it take for 240 mg of Californium to decay to 60 mg? *(Show a full algebraic solution for full marks.)*

$A = 240\left(\frac{1}{2}\right)^{t/45}$

$60 = 240\left(\frac{1}{2}\right)^{t/45}$

$\frac{1}{4} = \left(\frac{1}{2}\right)^{t/45}$

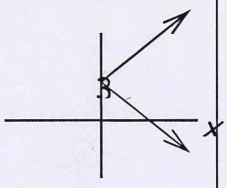
$\left(\frac{1}{2}\right)^2 = \left(\frac{1}{2}\right)^{t/45}$

$2 = \frac{t}{45}$

$90 = t$

∴ It would take 90 minutes.

30. Complete the following chart

Relation	Function? Yes/No	Domain	Range
a) $\{(4,-5),(6,-5),(0,7)\}$	Yes	$D:\{x=4,6,0\}$	$R:\{y=-5,7\}$
b) 	No	$D:\{x\in\mathbb{R}, x\geq 0\}$	$R:\{y\in\mathbb{R}\}$
c) $x^2+y^2=49$ $r=7$	No	$D:\{x\in\mathbb{R}, -7\leq x\leq 7\}$	$R:\{y\in\mathbb{R}, -7\leq x\leq 7\}$
d) $y=4x-7$	Yes	$D:\{x\in\mathbb{R}\}$	$R:\{y\in\mathbb{R}\}$
e) $y=-\sqrt{x-4}$	Yes	$D:\{x\in\mathbb{R}, x\geq 4\}$	$R:\{y\in\mathbb{R}, y\leq 0\}$
f) $y=\frac{1}{x}$	Yes	$D:\{x\in\mathbb{R}, x\neq 0\}$	$R:\{y\in\mathbb{R}, y\neq 0\}$

31. If $f(x)=5x-4$ and $g(x)=3x^2+7$, find:

a) $g(-2)$

$g(-2)=3(-2)^2+7$

$=3(4)+7$

$=12+7$

$=19$

b) $f(3a)$

$f(3a)=5(3a)-4$

$=15a-4$

c) the value of "*x*" when $f(x)=-39$

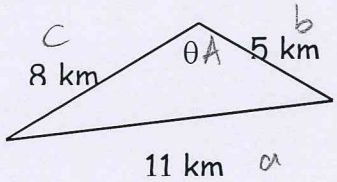
$y=-39$

$-39=5x-4$

$-35=5x$

$-7=x$

32. A marathon race follows a triangular course. The 3 legs of the race are 5 km, 11 km and 8 km long. Find the angle ϑ between the starting and finishing leg.



cosine law

$a^2 = b^2 + c^2 - 2bc\cos A$

$11^2 = 5^2 + 8^2 - 2(5)(8)\cos A$

$32 = -80\cos A$

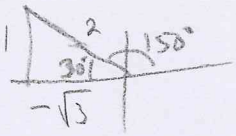
$\frac{32}{-80} = \cos A$

$\cos^{-1}\left(\frac{32}{-80}\right) = A$

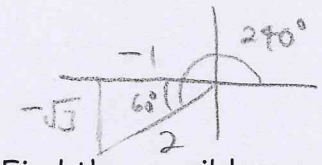
$113.6^\circ = A$

33. Find the exact value of each trigonometric ratio.

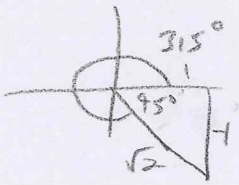
a) $\tan 150^\circ$ = $\frac{1}{-\sqrt{3}}$



b) $\sin 240^\circ$ = $-\frac{\sqrt{3}}{2}$

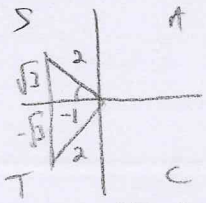


c) $\cos 315^\circ$ = $\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$



34. If $0^\circ \le A \le 360^\circ$, Find the possible measures of $\angle A$.

a) $\cos A = -\frac{1}{2}$



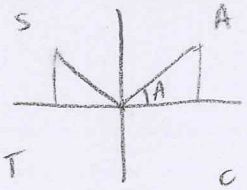
$\beta = 60^\circ$

$A = 180^\circ - 60^\circ = 120^\circ$

OR

$A = 180^\circ + 60^\circ = 240^\circ$

b) $\sin A = 0.7459$



$\sin A = 0.7459$

$A = \sin^{-1}(0.7459$

$A = 48.2^\circ$

OR

$A = 180^\circ - 48.2^\circ$

$= 131.8^\circ$

$A = 48.2^\circ, 131.8^\circ$

35. Graph $y = 3^x + 2$. Label at least 3 ordered pairs and state the equation of the asymptote.