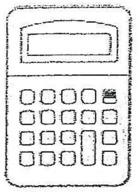

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MPM2D: Exam Review

**This package should be used as a study guide.*
This package does NOT contain every type of question we've done this semester. Study your old tests, quizzes and notes as well

1. Complete the following table, then determine the number of solutions of the linear systems below WITHOUT solving:

	Slopes (m)	y-intercepts (b)
One solution	different	Same or different
No solution	Same	different
Many solutions	Same	Same

a) $2x + y - 4 = 0 \rightarrow y = -2x + 4$
$x + 2y - 6 = 0 \rightarrow 2y = -x + 6$
$y = -\frac{x}{2} + 3$

the slopes are different,
∴ there is one solution

b) $2x + y = 6 \rightarrow y = -2x + 6$
$y = -2x + 8$

the slopes are the same, and the equations have different y-intercepts,
∴ there are no solutions

2. Find the intersection point of the lines below using SUBSTITUTION.

① $6x + 3y = 36$
② $2 + y = 5x \rightarrow y = 5x - 2$

Sub ② → ①
$6x + 3(5x - 2) = 36$
$6x + 15x - 6 = 36$
$21x = 42$
$x = \frac{42}{21}$
$x = 2$

Sub $x = 2$ → ②
$2 + y = 5(2)$
$y = 10 - 2$
$y = 8$

Check:
does $6(2) + 3(8) \stackrel{?}{=} 36$
$= 12 + 24$ | $= 36$
$= 36$ | $= 36$

Q.E.D.

∴ the intersection point is (2,8)

3. Find the intersection point of the lines below using ELIMINATION.

① $4x + 6y = 6$
② $(3x - 3y = 12)^{\times -2} \rightarrow -6x + 6y = -24$

$-6x + 6y = -24$
$-\ (4x + 6y = 6)$
$-10x + 0 = -30$
$x = \frac{-30}{-10}$
$\boxed{x = 3}$

Sub $x = 3$ → ①
$4(3) + 6y = 6$
$6y = 6 - 12$
$y = -\frac{6}{6}$
$\boxed{y = -1}$

Check:
does $3(3) - 3(-1) \stackrel{?}{=} 12$
$= 9 + 3$ | $= 12$
$= 12$ | $= 12$

Q.E.D.

∴ the P.O.I. is (3,-1)

P.O.I. (point of intersection)

4. A chemistry student was asked to make 100L of 48% alcohol solution by mixing 40% alcohol solution and 60% alcohol solution. How much of each solution must the student use?

let x = amount of 40% solution used
let n = amount of 60% solution used

① $x + n = 100 \rightarrow x = 100 - n$

② $0.4x + 0.6n = 48$

Sub ① → ②

$0.4(100-n) + 0.6n = 48$

$40 - 0.4n + 0.6n = 48$

$0.2n = 8$

$n = 40$

→ Sub $n = 40$ → ①

$x + 40 = 100$

$x = 60$

∴ the student must use 40L of 60% alchohol solution with 60L of 40% alcohol solution

→ check:

does $0.4(60) + 0.6(40) \stackrel{?}{=} 48$

$= 24 + 24$

$= 48$ ✓

5. A vending machine contains a total of 395 quarters and dimes. The total value of the coins is \$66.80. How many of each coin are there?

let n = the number of quarters
let x = the number of dimes

① $n + x = 395$

② $(0.25n + 0.1x = 66.8)^{\times 10} \rightarrow 2.5n + x = 668$

$$\begin{array}{rl} & 2.5n + x = 668 \\ - & n + x = 395 \\ \hline & 1.5n + 0 = 273 \end{array}$$

$n = \frac{273}{1.5}$

$\boxed{n = 182}$

→ Sub $n = 182$ → ①

$182 + x = 395$

$x = 395 - 182$

$\boxed{x = 213}$

∴ there are 182 quarters, and 213 dimes in the vending machine.

→ Check:

does $0.25(182) + 0.1(213) \stackrel{?}{=} 66.8$

$= 45.5 + 21.3$ | $= 66.8$

$= 66.8$ | $= 66.8$

Q.E.D.

6. For the line segment joining the points A(- 1, 5) and B(4, - 3) calculate:

x_1, y_1 x_2, y_2

a) the length of the line segment

$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$d=\sqrt{(4-(-1))^2+(-3-5)^2}$

$d=\sqrt{(5)^2+(-8)^2}$

$d=\sqrt{25+64}$

$d=\sqrt{89}$

$d=9.4339...$

$\therefore$ the distance is 9.43 units

b) the slope of the line segment

$m=\frac{\Delta y}{\Delta x}$

$m=(-3-5)\div(4-(-1))$

$m=-8\div 5$

$m=-\frac{8}{5}$

$\boxed{\frac{-8}{5}}$

$\therefore$ the slope of the line is $-\frac{8}{5}$

c) the midpoint of the line segment

$M=\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$M=\left(\frac{-1+4}{2}, \frac{5+(-3)}{2}\right)$

$M=\left(\frac{3}{2}, \frac{2}{2}\right)$

$M=(1.5, 1)$

$\therefore$ the midpoint of the line segment is (1.5, 1)

7. A communication tower located at the center of the grid sends out signals in a circle covered by the equation $x^2 + y^2 = 64$.

a) What is the radius of the circle?

$r^2=64$

$r=\sqrt{64}$

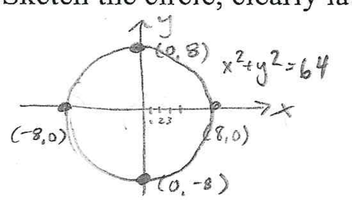
$r=8$

$\therefore$ the radius of the circle is 8

b) What is the centre of the circle?

the center of the circle is (0,0), which is the origin.

c) Sketch the circle, clearly labelling the x and y intercepts.



8. Given the vertices of ΔPQR, clearly **explain** how would you show that ΔPQR is a <u>right angle</u> <u>scalene</u> *triangle.*

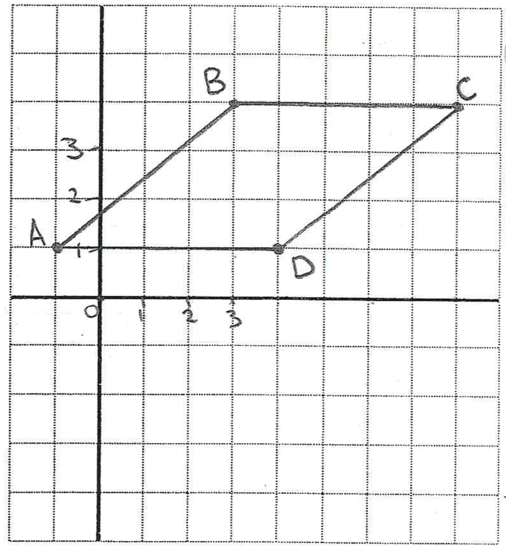
Firstly, to show that it is a right triangle, simply find the slopes of each side, and if one is the negative reciprocal of another, then it is. Secondly, to prove that the triangle is scalene, find the length of each side, and if they are all different, then it is.

9. Use the chart below to classify the quadrilateral PQRS. Explain your answer for full marks.

Side	PQ	QR	RS	PS
Slope	$\frac{-1}{2}$	2	$\frac{-1}{2}$	2
Length	3	3	3	3

The quadrilateral PQRS is a square. The sides that are opposite to each other are parallel (the have the same slopes), the sides intersect at 90° (2, and $-\frac{1}{2}$ are negative reciprocals), and every side is the same length (3 units).

10. a) A quadrilateral has vertices A(-1, 1), B(3, 4), C(8, 4), and D(4, 1). Verify that ABCD is a parallelogram.



Parallelograms have two pairs of opposite sides that are parallel.

Pair 1 - AD: $m=\frac{\Delta y}{\Delta x}$, $m=\frac{1-1}{4-(-1)}$, $m=0$

BC: $m=\frac{\Delta y}{\Delta x}$, $m=\frac{4-4}{8-3}$, $m=0$

lines AD, and BC are parallel.

Pair 2 - AB: $m=\frac{\Delta y}{\Delta x}$, $m=\frac{4-1}{3-(-1)}$, $m=\frac{3}{4}$

DC: $m=\frac{\Delta y}{\Delta x}$, $m=\frac{4-1}{8-4}$, $m=\frac{3}{4}$

lines AB and DC are also parallel, ∴ that means this shape is a parallelogram.

b) What would you have to do to prove that the quadrilateral is a rhombus?

In order to prove the quadrilateral was a rhombus, you would need to do everything shown above, and then prove that the length of each side is the same.

11. Given the quadratic equation y = -2(x + 5)(x +1)

a. State the zeros (**s** and **t**):

$x+5=0$ $\quad$ $x+1=0$

$x=-5$ $\quad$ $x=-1$

the zeros are (-5,0) and (-1,0)

b. Find the vertex of the function:

$y=-2(x+5)(x+1)$

$y=-2(x^2+6x+5)$

$y=-2x^2-12x-10$

$y=-2(x^2+6x)-10$

$y=-2(x^2+6x+9-9)-10$

$y=-2(x+3)^2-10+18$

$y=-2(x+3)^2+8$

∴ the vertex of the function is (-3, 8)

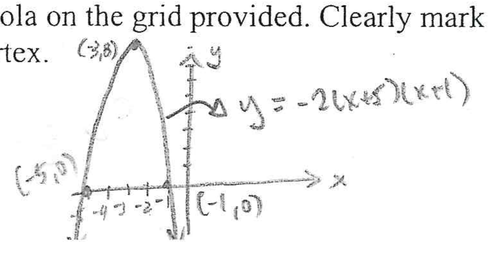
c. Is the vertex a minimum or a maximum?

The vertex is a maximum.

d. What is the axis of symmetry?

the axis of symmetry is $x=-3$

e. Sketch the parabola on the grid provided. Clearly mark the zeros and vertex.



12. A golf ball is hit off a tee. Its height above the ground can be modeled using the equation h = - *25t(t - 8)*, where h is the height of the ball in m, and t is the time in seconds.

a. When does the ball hit the ground?

$h=-25t(t-8)$

$-25t=0$ $\quad$ $t-8=0$

$t=0$ $\quad$ $t=8$

$\therefore$ the ball hits the ground after 8 seconds.

b. When does the ball reach the maximum height?

$h=-25t(t-8)$

$h=-25t^2+200t$

$h=-25(t^2-8t)$

$h=-25(t^2-8t+16-16)$

$h=-25(t-4)^2+400$

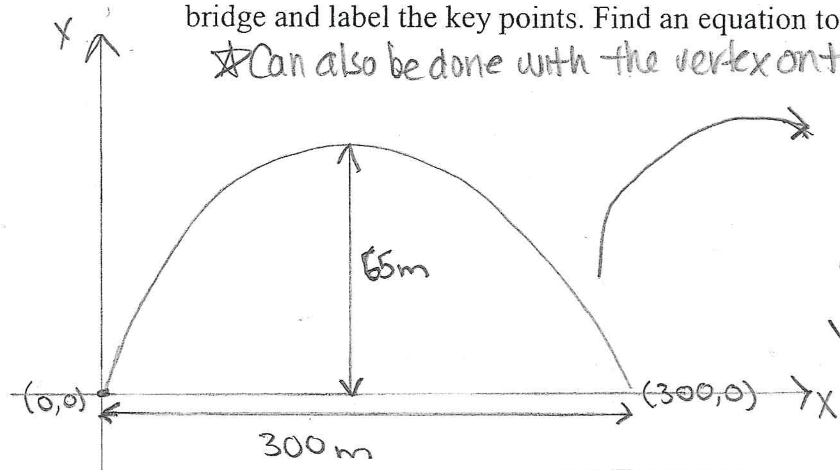
$\therefore$ the ball reaches the maximum height after 4 seconds

c. What is the maximum height of the ball?

from "b", I can conclude that the maximum height of the ball is 400 m above the ground.

13. Distance between the legs of Golden Gate Bridge in San Francisco is 300m and the arc is parabolic in shape. The maximum height of the arch is 65m. Sketch the bridge and label the key points. Find an equation to model the shape of the bridge.

☆Can also be done with the vertex on the y-axis → $\boxed{y=-\frac{13}{4500}x^2-195}$



$y=a(x-r)(x-s)$

$=a(x-0)(x-300)$

$=a(x)(x-300)$

vertex: $\frac{0+300}{2}$

$=150$

$\therefore$ the vertex is at $(150,65)$

Sub point into equation to find "a"

$65=a(150)(150-300)$

$0=a(150)(-150)-65$

$0=-22500a-65$

$22500a=-65$

$a=\frac{-65}{22500}$

$a=-\frac{13}{4500}$

$\therefore$ the equation that models the shape of this bridge is $y=-\frac{13}{4500}(x)(x-300)$

14. Expand and simplify:

a) $-3(x-2)(x+5)$

$=-3(x^2+3x-10)$

$=-3x^2-9x+30$

b) $(3x-8)^2$

$=(3x-8)(3x-8)$

$=9x^2-48x+64$

15. Factor each expression fully.

a) $x^2 - 2x - 15$

$=(x+3)(x-5)$

b) $3y^2 - 15y$

$=3y(y-5)$

c) $6w^2 - 7w + 2$

$=6w^2-3w-4w+2$

$=3w(2w-1)-2(2w-1)$

$=(2w-1)(3w-2)$

d) $2x^2 - 20x + 32$

$=2(x^2-10x+16)$

$=2(x-2)(x-8)$

e) $16b^2 - 9$

$=(4b+3)(4b-3)$

16. Solve (*find the roots*) each of the following equations:

a) $2x^2 - x - 15 = 0$

$0=2x^2-x-15$

$0=2x^2-6x+5x-15$

$0=2x(x-3)+5(x-3)$

$0=(2x+5)(x-3)$

$2x+5=0 \qquad x-3=0$

$2x=-5 \qquad x=3$

$x=-\frac{5}{2}$

$\therefore$ the roots of this equation are $-\frac{5}{2}$, and 3

b) Round your answer to 2 decimal places.

$2x^2 - 3x = 6$

$2x^2 - 3x - 6 = 0$

use quadratic formula:

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{3 \pm \sqrt{9+48}}{4}$

$x = \frac{3 \pm \sqrt{57}}{4}$ → $x = 2.64, -1.14$

∴ the roots of this equation are 2.64 and -1.14

17. Determine the **number** of roots of the following equations, show your work:

a) $3x^2 + 2x + 6 = 0$

$b^2 - 4ac$

$= 2^2 - 4(3)(6)$

$= 4 - 72$

$= -68$, so $b^2 - 4ac < 0$

∴ there are 2 complex roots, and 0 real roots

b) $(x + 5)^2 = 0$

$x^2 + 10x + 25 = 0$

$b^2 - 4ac$

$= 10^2 - 4(1)(25)$

$= 100 - 100$

$= 0$, so $b^2 - 4ac = 0$

∴ there is one root

c) $2x^2 - 14 = 0$

$b^2 - 4ac$

$= 0^2 - 4(2)(-14)$

$= 0 + 112$

$= 112$, so $b^2 - 4ac > 0$

∴ the equation has 2 real roots.

18. For each of the quadratic relations in the table, fill in the missing values.

Relation	Vertex	Axis of Symmetry	Direction of Opening (Up/Down)	Min/Max
$y = 2(x - 6)^2$	(6,0)	$x=6$	up	min
$y = -3(x + 7)^2 - 2$	(-7,-2)	$x=-7$	down	max
$y = 0.3x^2 - 20.5$	(0,-20.5)	$x=0$	up	min.

19. Write the following relations in ***vertex form*** and ***state the vertex***:

a) $y = x^2 - 20x + 5$

$y = x^2 - 20x + 100 - 100 + 5$

$y = (x-10)^2 - 95$

the vertex is $(10,-95)$

b) $y = -3x^2 + 12x - 8$

$y = -3(x^2 - 4x) - 8$

$y = -3(x^2 - 4x + 4 - 4) - 8$

$y = -3(x-2)^2 - 8 + 12$

$y = -3(x-2)^2 + 4$

the vertex is $(2,4)$

20. State the **vertex**, and all **transformations** of applied to $y = x^2$, then **graph**.
Show all steps.

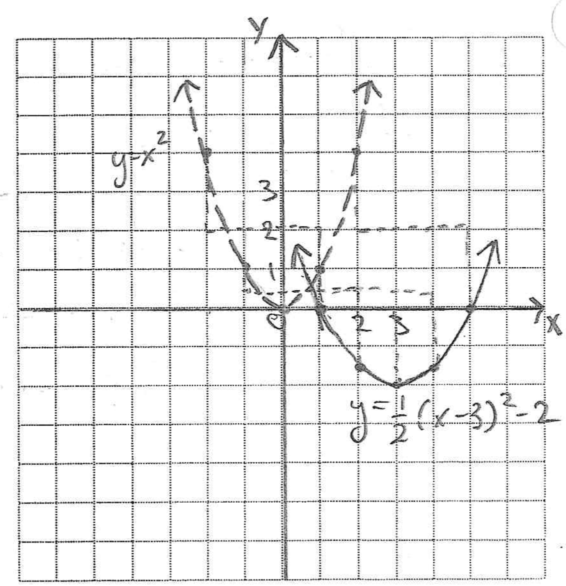
$$y = \frac{1}{2}(x-3)^2 - 2$$

Vertex:

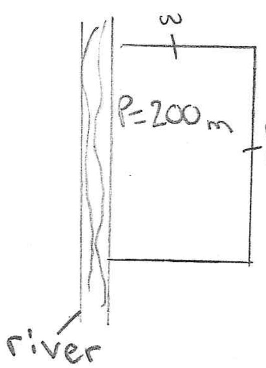
$(3,-2)$

Transformations:

- Vertically compress by a factor of $\frac{1}{2}$
- horizontal shift right by 3
- vertical shift down by 2



21. A farmer has 200m of fencing and wants to fence off a rectangular fence that borders a straight river. What dimensions of the field will give the maximum area?



let w= the width of the feild

let l= the length of the feild

$P = 2w + l$

$200 - 2w = l$

$A = lw$

$A = (200-2w)w$

$A = -2w^2 + 200w$

$A = -2(w^2 - 100w)$

$A = -2(w^2 - 100w + 2500 - 2500)$

$A = -2(w-50)^2 + 5000$

$w - 50 = 0$

$w = 50$

Sub into equation for P

$200 - 2(50) = l$

$l = 200 - 100$

$l = 100$

∴ the dimensions of the feild that give the maximum are are 50 m by 100 m.

22. A computer store sells 50 laptops every month at a price of $900 each. A recent survey shows that they can sell 2 more laptops for every $20 reduction in price. What price should they sell their computers at to maximize their revenue?

let x= number of $120 price reductions.

$R = (900-20x)(50+2x)$

$R = 45\,000 + 1800x - 1000x - 40x^2$

$R = -40x^2 + 800x + 45\,000$

$R = -40(x^2 - 20x) + 45\,000$

$R = -40(x^2 - 20x + 100 - 100) + 45\,000$

$R = -40(x-10)^2 + 45\,000 + 4000$

$R = -40(x-10)^2 + 49\,000$

$x - 10 = 0$

$x = 10$

Sub into price equation

$p = 900 - 20x$

$P = 900 - 20(10)$

$= 900 - 200$

$= 700$

∴ they should sell their computers for a price of $700.00 to maximize their revenue.

★ (You could also find the zeroes then calculate the midpoint, and sub this in to the price equation.)

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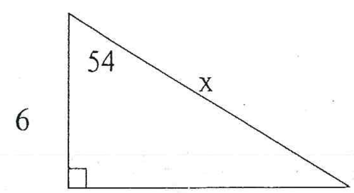
23. Solve for the indicated values:

a)



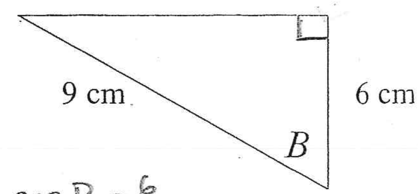
$\cos 54° = \frac{6}{x}$

$(\cos 54°)x = 6$

$x = \frac{6}{\cos 54°}$

$\boxed{x = 10.2 \text{ units}}$

b)

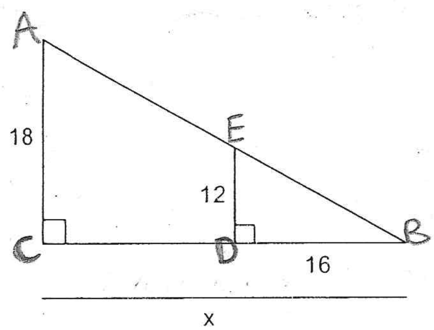


$\cos B = \frac{6}{9}$

$\angle B = \cos^{-1}\left(\frac{6}{9}\right)$

$\angle B = 48°$

c)



$\angle C = \angle EDB$ (90°)

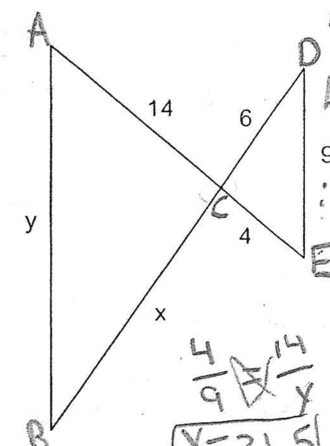
$\angle B = \angle B$ (same angle)

$\therefore \Delta ABC \sim \Delta EBD$

$\frac{x}{16} = \frac{18}{12}$

$\boxed{x = 24 \text{ units}}$

d)



$\angle ECD = \angle ACB$ (opposite angles)

$\angle EDC = \angle ABC$ ("Z" rule)

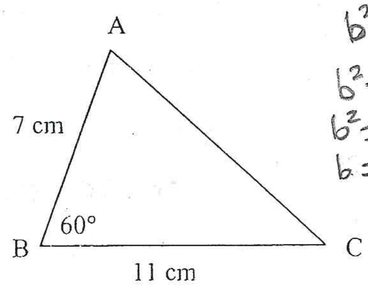
$\therefore \Delta ABC \sim \Delta EDC$

$\frac{4}{9} = \frac{14}{y}$ $\quad$ $\frac{9}{6} = \frac{31.5}{x}$

$\boxed{Y = 31.5}$ $\boxed{X = 21}$

units of length

24. Find b



$b^2 = a^2 + c^2 - 2ac \cos B$

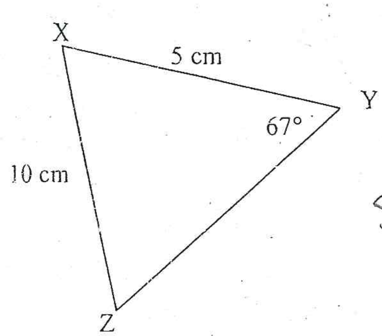
$b^2 = 11^2 + 7^2 - 2(11)(7) \cos 60°$

$b^2 = 170 - 154 \cos 60°$

$b = \sqrt{170 - 154 \cos 60°}$

$\boxed{b = 9.644 \text{ units}}$

25. Find angle Z:



$\frac{\sin Y}{y} = \frac{\sin Z}{z}$

$\frac{\sin 67°}{10} = \frac{\sin Z}{5}$

$\sin Z = 0.460\ldots$

$Z = (0.460\ldots)\sin^{-1}$

$\boxed{Z = 27.4°}$