MPM2D Feb 5 Park **Common Factoring** Unit Lesson 3

Recall: Find all the factors of 24. 1, 2, 3, 4, 6, 8, 12, 24

* Factor: number that you can divide the original number. Answer must be integers.

A polynomial is factored when it is written as a product of two or more polynomials. Factoring a polynomial is the reverse process of expanding or Ditributing.

term ↓ 2 or 3x

Eg. $7(x+2)$ is the factored form of $7x+14$

expanding →

← Factoring

Definitions:

Monomial is a polynomial with one term.

2 terms = Binomial

3 terms = Trinomial

Greatest Common Factor is the greatest number and/or variable that is a factor of two or more numbers or terms.

Examples:

a) The GCF of 12 and 8 is 4 — 2 is also common factor, but 4 > 2

b) The GCF of $12x^2y$ and $6xy^2$ is $6xy$ — 2, 6, 12, x, y, 6xy, xy

c) The GCF of $5xy^2$ and $20y^2$ is $5y^2$

To factor a polynomial:

1) Find the GCF
2) Write GCF outside of the brackets.
3) Divide the terms by the GCF

Example 1 Monomial common factor

Factor fully, if possible.

a) $6x+3$

① 3

② 3 ( )

③ 3 (2x + 1)

b) $8x^2 - 7x$

$8x^2 \div x^1 = 8x^1 = 8x$

① x

② x ( )

③ x (8x − 7)

c) $25k^6 + 15k^4$ 5, k, k^4

$5 \times 5 \times k^2 \times k^4 \rightarrow 5k^4$

① $5k^4$

② $5k^4$ ( )

③ $5k^4(5k^2+3)$

d) $21c^4d^3 - 28c^2d^5 + 7cd^3$ $d^2 \cdot d^3$

① $7cd^3$

② $7cd^3(3c^3d^{3-3} - 4cd^2 + 1)$

$= 7cd^3(3c^3 - 4cd^2 + 1)$

Challenge Questions (Preview):

Expand.

1. $(x+2)(x+3)$

$= x^2 + 3x + 2x + 6$

$= x^2 + 5x + 6$

2. $(x+2)(x^2+5x+7)$

$= x^3 + 5x^2 + 7x + 2x^2 + 10x + 14$

$= x^3 + 7x^2 + 17x + 14$

3. $4x^2y(3x^2+7xy^2)$

4. After expanding, you get $x^2 + 10x + 25$. What two binomials multiply to give you that trinomial?

Find 2 binomials that multiply to give you the polynomials below:

5. $x^2 + 9x + 8$ $\quad 9 = 8+1 \quad 8 = 8 \times 1$

$= x^2 + 8x + x + 8$

$= x(x+8) + (x+8) \longrightarrow (x+8)(x+1)$

$ax^2 + bx + c$

Prod: two integers $a \times c$

Add: " $= b$

6. $4x^2 - 9$ $\quad 4x^2 = 2x \times 2x \quad -9 = -3 \times 3$

$= (2x+3)(2x-3)$

7. $3x^2 + 13x + 12$

Super Challenge: Find 2 polynomials that multiply to give you the following polynomial:

8. $x^3 - 1$

e) $5x^5y^3 + 7w^5z^2$

Not factorable

f) $2x^3y^4z^2 + 4x^2y^2z^3$

$2, x, x^2, y^2, z^2$

① GCF $= 2x^2y^2z^2$

② $2x^2y^2z^2(1 \cdot x^{3-2} \cdot y^{4-2} \cdot z^{2-2} + 2x^{2-2} \cdot y^{2-2} \cdot z^{3-2})$

$= 2x^2y^2z^2(xy^2 + 2z)$

g) $x^2 + x$

① GCF $= x$

② $x(x+1)$

h) $8x^3 - 4x^2 + 32x$ $\quad 4 \times x = 4x$

① GCF $= 4x$

② $4x(2x^{3-1} - x^{2-1} + 8x^{1-1})$

$= 4x(2x^2 - x + 8)$

i) $3x^2y + xy^2$

j) $9a^2 - 8ab$

Example 2 Binomial Common Factor

Factor.

a) $3x(y+1) + 7z(y+1)$

① GCF $= (y+1)$

② $(y+1)\left[\frac{3x(y+1)}{(y+1)} + \frac{7z(y+1)}{(y+1)}\right]$

$= (y+1)(3x + 7z)$

b) $2x(x-3) - 5(x-3)$

$= (x-3)(2x-5)$

c) $5x^2(2x+3) - 10x(2x+3)$

① GCF = $5x(2x+3)$

② $5x(2x+3)(x-2)$

① $(2x+3)$

② $(2x+3)(5x^2 - 10x)$

$= 5x(2x+3)(x-2)$

Example 3 Factor by Grouping

Factor.

a) $ax + ay + 2x + 2y$

a 2

$= a(x+y) + 2(x+y) \longrightarrow GCF = (x+y)$

$= (x+y)(a+2)$

HW b) $9x^2 + 15x + 3x + 5$

$= 3x(3x+5) + (3x+5)$

$= (3x+5)(3x+1)$

HW **Challenge** Factor.

a) $xy + x + y + 1$