

$$c) 5x^2(2x+3) - 10x(2x+3)$$

$$\textcircled{1} \text{ GCF} = 5x(2x+3)$$

$$\textcircled{2} 5x(2x+3)(x-2)$$

$$\textcircled{1} (2x+3)$$

$$\textcircled{2} (2x+3)(5x^2 - 10x)$$

$$= 5x(2x+3)(x-2)$$

### Example 3 Factor by Grouping

Factor.

$$a) (ax + ay) + (2x + 2y)$$

$a$        $2$

$$= a(x+y) + 2(x+y) \rightarrow \text{GCF} = (x+y)$$

$$= (x+y)(a+2)$$

$$\text{HW b) } 9x^2 + 15x + 3x + 5$$

$$= 3x(3x+5) + (3x+5)$$

$$= (3x+5)(3x+1)$$

HW Challenge Factor.

$$a) xy + x + y + 1$$

$$= x(y+1) + (y+1) \rightarrow \text{common factor: } (y+1)$$

$$= (y+1)(x+1)$$

$$e) 5x^5y^3 + 7w^5z^2$$

Not factorable

$$2, x, x^2, y^2, z^2$$

$$f) (2x^3y^4z^2) + (4x^2y^2z^3)$$

$$\textcircled{1} \text{ GCF} = 2x^2y^2z^2$$

$$\begin{aligned}\textcircled{2} \quad & 2x^2y^2z^2(1 \cdot x^{3-2} \cdot y^{4-2} \cdot z^{2-2}) + 2x^2y^2z^2 \\ & = 2x^2y^2z^2(xy^2 + 2z)\end{aligned}$$

$$g) x^2 + x$$

$$h) 8x^3 - 4x^2 + 32x \quad 4x \cancel{x} = 4x$$

$$\textcircled{1} \text{ GCF} = x$$

$$\textcircled{1} \text{ GCF} = 4x$$

$$\textcircled{2} \quad x(x+1)$$

$$\begin{aligned}\textcircled{2} \quad & 4x(2x^{3-1} - x^{2-1} + 8x^{1-1}) \\ & = 4x(2x^2 - x + 8)\end{aligned}$$

$$i) 3x^2y + xy^2$$

$$j) 9a^2 - 8ab$$

### Example 2 Binomial Common Factor

Factor.

$$a) 3x(y+1) + 7z(y+1)$$

$$\textcircled{1} \text{ GCF} = (y+1)$$

$$\textcircled{2} \quad (y+1) \left[ \frac{3x(y+1)}{(y+1)} + \frac{7z(y+1)}{(y+1)} \right]$$

$$= (y+1)(3x + 7z)$$

$$b) 2x(x-3) - 5(x-3)$$

$$= (x-3)(2x-5)$$

$$\begin{aligned}
 \text{HW b) } & (4ax + 2by - 8ay - bx) \\
 & = 4a(x - 2y) - b(2x - 2y) \\
 & = (x - 2y)(4a - b)
 \end{aligned}$$

\* Why did I choose " $-b$ " instead of " $b$ " as a common factor?  
 Because I wanted the terms in the second bracket to be same as the terms in the first bracket.

### Summary:

Factoring a polynomial is the opposite of expanding a polynomial.

$$\begin{array}{c}
 \text{Factoring} \\
 \curvearrowleft x^2 + 3x = x(x + 3) \\
 \curvearrowright \text{expanding}
 \end{array}$$

### Practice:

From the website <http://www.math-worksheet.org/common-factor-only>  
 (Also see website for videos)

For all questions below, factor.

### HW Basic Difficulty

1)  $-8v + 10v^2 = 2v(5v - 4)$

2)  $-x - x^2 = -x(x + 1)$

3)  $4b - 4 = 4(b - 1)$

4)  $6n^2 - 9 = 3(2n^2 - 3)$

5)  $16p^4 + 4p^6 = 4p^4(4p + p^2)$

6)  $15a^2 - 10a = 5a(3a - 2)$

7)  $-4k^2 + 8 = -4(k^2 - 2)$

8)  $25m^3 + 5m = 5m(5m^2 + 1)$

### HW Medium Difficulty

1)  $32v^6 + 8vu - 80v^2 = 8v(4v^5 + u - 10v)$

2)  $6yx^5 + 2y^3x - 6y = 2y(3x^5 + y^2x - 3)$

3)  $80v^2u - 8v^3 + 40v^2 = 8V^2(10u - v + 5)$

4)  $-12x + 6xy^2 - 15x^3y^3 = -3x(4 - 2y^2 + 5x^2y^3)$

5)  $-4xy^7 - 10x^4y^2 - 6xy^3$

6)  $-9x^2y^3 - 3xy^3 - 4x^2y^2$

$$\begin{aligned}
 & = -2x(2y^7 + 5x^2y^2 + 3y^3) \\
 & = -x(9x(y^3 + 3y^3 + 4xy^2))
 \end{aligned}$$

# HW Hard Difficulty

1)  $-4x^5y + 20x^4yz - 24x^3y^3 + 28x^3yz$

2)  $6j^2k^4h^3 + 60j^3k^6 + 36j^5k^4h - 36j^3k^2$

3)  $90y^8z^7x - 18y^9z^5 + 9y^7z^5x - 27y^6z^5$

4)  $7pq^3r^6 - 14p^2q^3 - 14pq^3r + 63pq^3$

HW From the website <http://www.math-worksheet.org/by-grouping>  
 (Also see website for videos)

1)  $5v^3 - 2v^2 + 25v - 10$

2)  $15x^3 - 25x^2 + 12x - 20$

3)  $2b^3 + b^2 + 8b + 4$

4)  $8k^3 - 6k^2 + 4k - 3$

5)  $3a^3 + 15a^2 + 2a + 10$

6)  $20p^3 + 5p^2 + 8p + 2$

$= 3a^2(a+5) + 2(a+5)$

$= 5p^2(4p+1) + 2(4p+1)$

$= (a+5)(3a^2+2)$

$= (4p+1)(5p^2+2)$

## Answers:

### Easy

1)  $2v(-4 + 5v)$   
 5)  $4p^4(4 + p^2)$

2)  $-x(1+x)$   
 6)  $5a(3a-2)$

3)  $4(b-1)$   
 7)  $4(-k^2+2)$

4)  $3(2n^2-3)$   
 8)  $5m(5m^2+1)$

### Medium

1)  $8v(4v^5 + u - 10v)$   
 4)  $3x(-4 + 2y^2 - 5x^2y^3)$

2)  $2y(3x^5 + xy^2 - 3)$   
 5)  $-2xy^2(2y^5 + 5x^3 + 3y)$

3)  $8v^2(10u - v + 5)$   
 6)  $-xy^2(9xy + 3y + 4x)$

### Hard

1)  $4x^3y(-x^2 + 5xz - 6y^2 + 7z)$   
 3)  $9y^6z^5(10xy^2z^2 - 2y^3 + xy - 3)$

2)  $6j^2k^2(h^3k^2 + 10jk^4 + 6hjk^2 - 6j)$   
 4)  $7pq^3(r^6 - 2p - 2r + 9)$

## Factor by Grouping

1)  $(v^2 + 5)(5v - 2)$   
 5)  $(3a^2 + 2)(a + 5)$

2)  $(5x^2 + 4)(3x - 5)$   
 6)  $(5p^2 + 2)(4p + 1)$

3)  $(b^2 + 4)(2b + 1)$

4)  $(2k^2 + 1)(4k - 3)$

## Self-Quiz

Expand. (Remember this is the opposite of factoring!)

a)  $5(x + 4)$

b)  $2x(3x - 5)$

c)  $-7(x - 2)$

d)  $(x + 1)(2x - 3)$

e)  $2(x - 3)(x + 1)$

Factor.

f)  $3x^2 - 6x$

g)  $4x^2y^3 + 8xy^5 - 12x^6y^3$

$$\begin{aligned} h) & \quad 6m^3 + 10m^2 + 3m - 5 \\ & = 2m^2(3m^1 - 5) + (3m - 5) \\ & = (3m - 5)(2m^2 + 1) \end{aligned}$$

### Answers

a)  $5x + 20$

b)  $6x^2 - 10x$

c)  $-7x + 14$

d)  $2x^2 - x - 3$

e)  $2x^2 - 4x - 6$

f)  $3x(x - 2)$

g)  $4xy^3(x + 2y - 3x^5)$

h)  $(2m^2 + 1)(3m - 5)$

Factoring Hard Trinomials - Decomposition

**Recall:** Factor fully.

a)  $4x^2 - 8x - 32$

$$= 4(x^2 - 2x - 8)$$

b)  $3x(x + 8) + 5(x + 8)$

$$= (x+8)(3x+5)$$

**Warm up:**

Find two integers with the given product and sum

Product = 45, sum = 14      Integers:

Integers	Product	Sum
1, 45	45	46 X
9 × 5	(45)	(14)

Product = 6, sum = -5      Integers:

Integers	Product	Sum
-2, 3	6	5 X
-2, -3	(6)	(-5)

Product = -10, sum = 3      Integers:

Integers	Product	Sum

Product = -20, sum = -8      Integers:

Integers	Product	Sum
5 × -4	-20	1 X
-10, 2	(-20)	(-8)

**Definition:** A hard trinomial is of the form  $ax^2 + bx + c$ .       $a, b, c = \text{integers}$

Expand  $(2x + 3)(3x + 4)$  algebraically and using an area diagram.

When we are factoring, we are trying to reverse this process.

$$ax^2 + bx + c$$

To factor a hard trinomial by decomposition

- 1) Always look for the common factor first when factoring a trinomial
- 2) Find two integers whose product is  $ac$  and whose sum is  $b$ .
- 3) Then break up the middle term (decompose!)
- 4) Factor by grouping.

$$\hookrightarrow = bx$$

**Example 1** Factor, if possible.  $c = 4$

a)  $3x^2 + 8x + 4$        $a = 3$   
① none       $ax^2 + bx + c$        $b = 8$

②  $ac = 3 \times 4 = 12 \rightarrow$

$b = 8$

$1 \times 12 = 12$   
 $2 \times 6 = 12$

$1 \times 12 = 12$   
 $2 \times 6 = 12$

③  $3x^2 + 2x + 6x + 4$

④  $= 3x(x+2) + 2(x+2)$

$= (x+2)(3x+2)$

b)  $3x^2 + 2x + 4$

① none

②  $ac = 3 \times 4 = 12 \rightarrow 1 \times 12 = 12 \times$

③  $b = 2$

$2 \times 6 = 12 \times$

④  $3 \times 4 = 12 \times$

∴ Not factorable

c)  $6x^2 - 5x + 1$

$ax^2 + bx + c$   
 $a = 6$   
 $b = -5$

① none

$c = 1$

③  $6x^2 - 2x - 3x + 1$

④  $= 6x^2 - 3x - 2x + 1$

②  $ac = 6 \times 1 = 6 \rightarrow 1 \times 6 (\times)$

$b = -5$

$2 \times 3 (\times)$   
 $(-2) \times (-3) \checkmark$

$= 3x(2x-1) - (2x-1)$

$= (2x-1)(3x-1)$

**Example 2** Trinomials with Two Variables

Factor  $10x^2 - 3xy - 4y^2$        $a = 10$

$ax^2 + bx + c$        $b = -3y$

$ac = 10 \times -4y^2 = -40y^2$        $c = -4y^2$

$b = -3y$

$\begin{matrix} 10y & -4y \\ \times & \end{matrix}$

$(5)y(-8)y$

$\begin{matrix} 10x^2 & -3xy \\ + & \end{matrix}$

$-8xy - 4y^2$

$\Rightarrow (2x+y)(5x-4y)$

$\therefore (2x+y)(5x-4y)$

③

④

$10x^2 + 5xy - 8xy - 4y^2$

$= 5x(2x+y) - 4y(2x+y)$

### Example 3 Remove a Common Factor

$$\begin{aligned} \textcircled{1} \quad & 2 \\ = & 2(8x^2 + 13x - 6) \\ \Rightarrow \textcircled{2} \quad & ac = 8 \times (-6) = -48 \\ b = 13 & \quad \downarrow \\ -6 \times 8 & \Rightarrow 2 \times \\ -4 \times 12 & \Rightarrow 8 \times \\ -3 \times 16 & \checkmark \end{aligned}$$

\textcircled{3}

$$\begin{aligned} & = 2(8x^2 - 3x + 16x - 6) \\ \textcircled{4} = & \quad \cancel{8x} \quad \cancel{-3} \\ & = 2[8x(x+2) - 3(x+2)] \\ & = 2[(x+2)(8x-3)] \\ & = 2(x+2)(8x-3) \end{aligned}$$

### Example 4 Simple Trinomials

Simple trinomials are of the form  $x^2 + bx + c$ , so  $a = 1$

HW

Factor.

$$a = 1$$

$$\text{a)} \quad x^2 - 4x - 21 \quad b = -4$$

$$c = -21 \rightarrow -7 \times 3$$

$$\begin{aligned} & = x^2 - 7x + 3x - 21 \\ & = x(x+3) - 7(x+3) \rightarrow = (x+3)(x-7) \end{aligned}$$

$$\text{b)} \quad x^2 - 29x + 28$$

$$a = 1$$

$$7 \times 4 \quad (\times)$$

$$\text{③) } x^2 - 28x - x + 28$$

$$b = -29$$

$$= x(x-1) - 28(x-1)$$

$$c = 28$$

$$14 \times 2 \quad (\times)$$

$$= (x-1)(x-28)$$

$$\text{c)} \quad x^2 + 3x - 18 \quad 28 \times -1 \quad (\checkmark)$$

$$a = 1$$

$$\text{③) } x^2 + 6x - 3x - 18$$

$$b = 3$$

$$= x(x-3) + 6(x-3)$$

$$c = -18$$

$$= (x-3)(x+6)$$

\*Do you see a shortcut? Feel free to use this on quizzes and tests!

### Summary:

Always look for a common factor first when factoring a trinomial.

To factor  $ax^2 + bx + c$ , find two integers whose product is  $ac$  and whose sum is  $b$ . Then, break up the middle term and factor by grouping.

Not all quadratic expressions of the form  $ax^2 + bx + c$  will be factorable.