Feb 17 Park

MCR3U

Equivalent Expressions

Example 1 Show that the following expressions are equivalent:

(A) $\frac{5}{m^2+mn} - \frac{2}{m^2-mn} + \frac{3m-2n}{m^3-mn^2}$ $\quad m(m^2-n^2)$

$\frac{6m-9n}{2m^2+3mn} \div \frac{m^2-n^2}{2m+3n}$ (B)

Note: You need to show algebraically that they are equal. Subbing in points is not enough, some points might be shared, but that does not show that all points are equivalent.

* Key Concepts (for Equivalent Expressions)

① To determine if two expressions are equivalent, simplify both to see if they are algebrically the same.

② Check the rational expressions (original or factored form of original) for restrictions by determining where the denominator is zero

(A) Left side

$\frac{5}{(m+n)} - \frac{2}{m(m-n)} + \frac{3m-2n}{m(m+n)(m-n)}$

* CD $= m(m+n)(m-n)$, $m \neq 0$, $m \neq n$, $m \neq -n$

$= \frac{5(m-n) - 2(m+n) + 3m - 2n}{m(m+n)(m-n)}$

$= \frac{5m - 5n - 2m - 2n + 3m - 2n}{m(m+n)(m-n)}$

$= \frac{6m - 9n}{m(m+n)(m-n)}$

$= \frac{3(2m-3n)}{m(m+n)(m-n)}$, $m \neq 0$, $m \neq n$, $m \neq -n$

(B) Right side

$= \frac{6m-9n}{2m^2+3mn} \times \frac{2m+3n}{(m+n)(m-n)}$

$= \frac{3(2m-3n)}{m(2m+3n)} \times \frac{(2m+3n)}{(m+n)(m-n)}$

* Restriction → $m \neq 0$, $2m+3n \neq 0$

→ $m \neq n$ → $2m \neq -3n$

→ $m \neq -n$ $\quad m \neq \frac{-3n}{2}$

$= \frac{3(2m-3n)}{m(m+n)(m-n)}$,

$\therefore$ LS = RS except $m \neq -\frac{3n}{2}$

= same identity

When two expressions are equivalent, for example $2x$ is equal to $\frac{4x}{2}$, the statement $2x = \frac{4x}{2}$ is called an **identity**.

Example 2 Prove the identity below, and state restrictions.

$$\text{(A)} \rightarrow \frac{x^2-2x-8}{x^2-x-12} = \frac{x^2-x-6}{x^2-9} \leftarrow \text{(B)}$$

(A) $\frac{(x-4)(x+2)}{(x-4)(x+3)}$

$= \frac{(x+2)}{(x+3)}$, $x \neq 4$, $x \neq -3$

(B) $= \frac{(x-3)(x+2)}{(x+3)(x-3)}$

$= \frac{(x+2)}{(x+3)}$, $x \neq 3$, $x \neq -3$

∴ These two expressions are equal (or same identity) except when $x = 3$ and $x = 4$

Homework: pg. 83 #C1, C2, C3, 3, 4, 5EOO, 6bdf,8, 9, 15*, 16*