Feb 10

MPM2D

Difference of Squares

Feb 17: Unit Test

Recall: Find the squares of the numbers 1 through 12.
(*You should be getting good at this by now!*)

$1^2 =$	$4^2 =$	$7^2 =$	$10^2 =$
$2^2 =$	$5^2 =$	$8^2 =$	$11^2 =$
$3^2 =$	$6^2 =$	$9^2 =$	$12^2 =$

Difference of Squares

Investigate:
Expand and Simplify

a) $(x+1)(x-1)$

$= x^2 - x + x - 1$

$= x^2 - 1$

b) $(3c-10)(3c+10)$

c) $(2m-4)(2m+4)$

$= 4m^2 + 8m - 8m - 16$

$= 4m^2 - 16$

d) $(a+b)(a-b)$

$= a^2 - ab + ab - b^2$

$= a^2 - b^2$

What patterns do you notice?

$\boxed{a^2 - b^2 = (a+b)(a-b)}$ = Differences of squares

To factor difference of squares: _____

Example 1 Factor.

a) $x^2 - 25$ $\quad a = x$, $b = 5$

$= x^2 - 5^2$

$= (x+5)(x-5)$

b) $y^2 - 36$ $\quad a = y$, $b = 6$

$= y^2 - 6^2$

$= (y+6)(y-6)$

c) $16k^2 - 49$ $\quad a = 4k$, $b = 7$

$= (4k)^2 - 7^2$

$= (4k+7)(4k-7)$

d) $25n^2 - 144$ $\quad a = 5n$, $b = 12$

$= (5n)^2 - 12^2$

$= (5n+12)(5n-12)$

$* \ (4k)^2 = 4^2 \cdot k^2 = 16k^2$

What value of k would make the following polynomials perfect square trinomials?

$\Downarrow$

$a^2 + 2ab + b^2$ or

$a^2 - 2ab + b^2$

a) $x^2 - 4x + k$

$a = x$

$b = ?$

$-\frac{2 \cdot x \cdot b}{-2x} = \frac{-4x}{-2x}$

$2ab = -4x$

$\therefore b = 2$

$k = b^2 =$

$\Rightarrow \therefore k = 4$

b) $4x^2 + kx + 25$

$a = 2x$

$b = 5$

$= (2x)^2 + (2 \cdot 2x \cdot 5) + 5^2$

$= 4x^2 + 20x + 25 \longrightarrow k = 20$

c) $100x^2 + kx + 81 \rightarrow k = ?$

$a = 10x$ $= (10x)^2 + (2 \cdot 10x \cdot 9) + 9^2$

$b = 9$ $kx = 180x$ $\therefore k = 180$

d) $49x^2 - 42x + k$

$a = 7x$ $(7x)^2 - 2 \cdot (7x) \cdot b + b^2$

$b = ?$ $-42x = -14x b$

$\frac{-42x^1}{-14x^1} = b$

$\therefore b = 3$

$k = b^2 = 9$

Create two trinomials that are **not** perfect square trinomials and therefore are not factorable!

① $3x^2 + 35x + 5$

② $25x^2 + 17x + 36$

Review: Identify the type of polynomial. Factor, if possible.

HW

a) $5x^2 - 8x - 4$ $ac = -20$, $b = -8$

$(-10) \times 2$

$= 5x^2 - 10x + 2x - 4$

$= 5x(x-2) + 2(x-2)$

$= (x-2)(5x+2)$

b) $x^2 + 2x + 1$

$a = x$

$b = 1$

$= x^2 + (2 \cdot x \cdot 1) + 1^2$

$= (x+1)^2$

c) $8x^3y + 4x^2y - 6xy^5$

$= 2xy(4x^2 + 2x - 3y^4)$ → This is not in the form of hard trinomial

$ax^2 + bx + c$

d) $3x^2 - 3x + 9$

$= 3(x^2 - x + 3)$

$ac = 1 \times 3 = 3$

$b = -1$

I can't find any two integers, which satisfy two conditions.

$\therefore 3(x^2 - x + 3)$

e) $25x^2 - 20x + 4$

$a = 5x$

$= (5x)^2 - (2 \cdot 5x \cdot 2) + 2^2$ $b = 2$

$= (5x - 2)^2$

f) $2x^2 - 12x + 18$

$= 2(x^2 - 6x + 9)$

$a = x$

$= 2(x^2 - (2 \cdot x \cdot 3) + 3^2)$ $b = 3$

$= 2(x-3)^2$

* HW: Read "HW List made on Feb 9"

Example 2 Factor, if possible

a) $x^2 - 100$

$a = x$

$= x^2 - 10^2$ $\quad b = 10$

$= (x+10)(x-10)$

b) $98a^2 - 450b^2$

$= 2(49a^2 - 225b^2)$ $\quad a = 7a$

$= 2[(7a)^2 - (15b)^2]$ $\quad b = 15b$

$= 2[(7a+15b)(7a-15b)]$

$= 2(7a+15b)(7a-15b)$

c) $25x^2 + 49$

$= \cancel{(5x)^2 - (-7)^2}$

$= 25x^2 + 49$

d) $3x^2 - 27y^2$

$= 3(x^2 - 9y^2)$ $\quad a = x$

$= 3(x^2 - (3y)^2)$ $\quad b = 3y$

$= 3(x+3y)(x-3y)$

Example 3 Create a difference of squares question. See if your classmates can factor it.

$64a^2 - 169b^2 = (8a)^2 - (13b)^2 = (8a+13b)(8a-13b)$

***Students often get confused between perfect squares and difference of squares**

Create a perfect square trinomial and create a difference of squares using the terms $3x$ and $4y$.

Perfect Square Trinomial:

$(3x+4y)(3x+4y)$

$= 9x^2 + 12xy + 12xy + 16y^2$

$= 9x^2 + 24xy + 16y^2$

Difference of Squares: $a = 3x$, $b = 4y$

$(3x+4y)(3x-4y)$

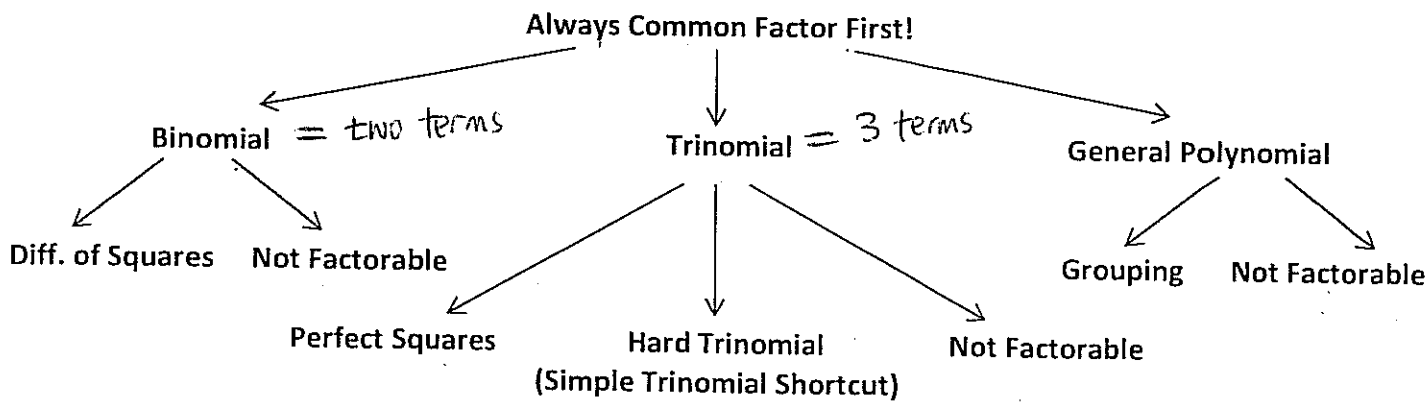
$= 9x^2 - 16y^2$

What is the difference between perfect squares trinomials and difference of squares?

* Perfect Square T has 3 terms whereas
Differences of Square // 2 //

You have now learned all the basic types of factoring! **(Yay!)**
The hard part is distinguishing between the different types on a test. ☹

How to decide which type of factoring you need: ☺



Review Self Quiz: Identify the type of factoring. Factor, if possible.

a) $x^2 - 3x - 40$

$ac = 1 \times -40 = -40$

$b = -3$

$(-8) \times 5$

$= (x-8)(x+5)$

b) $25x^2 - 36y^2$

$= (5x)^2 - (6y)^2$

$= (5x+6y)(5x-6y)$

c) $2x^2 - 2x - 24$

$ac = 1 \times -12 = -12$

$b = -1$

$(-4) \times 3$

$= 2(x^2 - x - 12)$

$= 2(x-4)(x+3)$

d) $144x^2 + 96x - 16$

$= 4(36x^2 + 96x - 4)$

$= 16(9x^2 + 24x - 1)$

$ac = -9$

$b = 24$

No two integers will satisfy

e) $6x^2 - 7x - 3$

$ac = 6 \times -3 = -18$

$b = -7$

$(-9) \times 2$

$= 6x^2 - 9x + 2x - 3$

$= 3x(2x-3) + (2x-3)$

$= (3x+1)(2x-3)$

f) $9x^2 + 64$

$\Rightarrow$ Not factorable

HW Feb 10

Challenge

a) $4m^2 - (6m-7)^2$

$a = 2m$

$b = 6m - 7$

$= (2m)^2 - (6m-7)^2$

$= (2m + 6m - 7)(2m - (6m-7))$

$= (8m-7)(-4m+7) = -(8m-7)(4m-7)$

b) $(5m+2)^2 - (3m-8)^2$

$a = 5m + 2$

$b = 3m - 8$

$= (5m + 2 + 3m - 8)(5m + 2 - (3m - 8))$

$= (8m - 6)(2m + 10)$

$= 2(4m-3) \cdot 2(m+5)$

$= 4(4m-3)(m+5)$

c) $9(2a+5b)^2 - 4(7a-3b)^2$

$= [3(2a+5b)]^2 - [2(7a-3b)]^2$

$= [6a+15b]^2 - [14a-6b]^2$

$a = 6a + 15b$

$b = 14 - 6b$

$= (6a + 15b + 14 - 6b)(6a + 15b - (14 - 6b))$

d) $a^2 - b^2 + 8bc - 16c^2$

$= (6a + 9b + 14)(6a + 21b - 14)$

e) $25 - m^2 - 12mn - 36n^2$

f) $x^2 - a^2 - y^2 - 2ay$

g) $x^2 + 9y^2 - 25z^2 - 6xy$

h) $x^3 + x^2 - x - 1$

i) $a^{2n} - b^{2n}$

j) $4x^4 + 35x^2 + 49$