* Tues (Feb 24) : Unit Test

* $\mathbb{R}$ = Real numbers
= rational # + irrational #

MCR3U / Feb 18
Mr. Park

Radicals

* $\mathbb{R}$ > Rational # > Integers > whole # > Natural #

Rational numbers are numbers that can be written in fraction form $\frac{a}{b}$, a and b are integers and $b \neq 0$

Rational numbers include:

- terminating decimals eg. $\frac{1}{2} = 0.5$ e.g $3 = \frac{3}{1}$
- periodic decimals eg. $\frac{3}{11} = 0.2727\overline{27}$

Irrational Numbers are numbers which can't be written in fraction form. Irrational numbers include all non-terminating, non-repeating decimals.

Index ↖ $\sqrt[2]{3}$ → radicand

eg. π, radicals ($\sqrt{3}$, $\sqrt{11}$), some trig ratios eg. sin 50°

Radicals: $\sqrt{3}$, $\sqrt[5]{28}$, $\sqrt{4} = \sqrt{2^2} = 2$ * $\sqrt[3]{8} = \sqrt[3]{(2)^3} = 2$

Radicand: 3 , 28 (number under the root sign)

↳ It is not a radical #.

Entire Radical $\sqrt{24} = \sqrt{6 \times 4}$ (the entire number is under the root sign)

↳ similar to improper fraction

Mixed Radicals $2\sqrt{6} = 2 \cdot \sqrt[2]{6}$ (radical is multiplied by a number)

↳ similar to mixed fraction

rational # × radical

Example 1 Change entire radicals to mixed radicals

a) $\sqrt{24}$

$= \sqrt{4 \times 6}$

$= (\sqrt{4})(\sqrt{6})$

$= (\sqrt[2]{2^2}) \cdot \sqrt{6} = 2\sqrt{6}$

b) $\sqrt{98}$

$= \sqrt{2 \cdot (49)}$

$= \sqrt{2}(\sqrt{49})$

$= \sqrt{2} \cdot \sqrt[2]{7^2}$

$= 7\sqrt{2}$

c) $\sqrt{50}$

$= (\sqrt{2})(\sqrt{25})$

$= \sqrt{2} \cdot \sqrt{5^2} = 5\sqrt{2}$

d) $\sqrt{180}$

$= \sqrt{4 \cdot 9 \cdot 5}$

$= (\sqrt{4})(\sqrt{9})(\sqrt{5})$

$= (\sqrt{2^2}) \cdot (\sqrt{3^2})(\sqrt{5})$

$= 2 \cdot 3 \cdot \sqrt{5}$

$= 6\sqrt{5}$

Example 2 Adding and subtracting radicals

Just as in algebra where like terms can be combined, like radicals can also be combined.

a) $5\sqrt{3}+4\sqrt{3}$

$= 9\sqrt{3}$

b) $\boxed{4\sqrt{6}}-3\sqrt{5}\overset{+}{+}\boxed{2\sqrt{6}}+7\sqrt{5}$

$= 6\sqrt{6} + 4\sqrt{5}$

b) $2\sqrt{20}-\sqrt{125}+2\sqrt{45}+6\sqrt{5}$

$= 2\cdot(\sqrt{4})\cdot(\sqrt{5}) - (\sqrt{5})(\sqrt{25}) + 2(\sqrt{5})(\sqrt{9}) + 6\sqrt{5}$

$= (2\cdot 2\cdot\sqrt{5}) - 5\sqrt{5} + (2\cdot 3\cdot\sqrt{5}) + 6\sqrt{5}$

$= 4\sqrt{5} - 5\sqrt{5} + 6\sqrt{5} + 6\sqrt{5}$

$= 11\sqrt{5}$

Example 3 Multiplying Radicals

a) $\sqrt{2}\times\sqrt{6}$

$= \sqrt{2\cdot 6}$

$= \sqrt{12}$

$= (\sqrt{4})\cdot(\sqrt{3})$

$= \sqrt{2^2}\cdot\sqrt{3} = 2\sqrt{3}$

b) $(3\sqrt{2})(2\sqrt{5})$

$= 3\cdot 2\cdot\sqrt{2\cdot 5}$

$= 6\sqrt{10}$

b) $2\sqrt{3}\,(\overset{\textcircled{1}}{\sqrt{5}}-\overset{\textcircled{2}}{2})$

$= \overset{\textcircled{1}}{2\sqrt{3\cdot 5}} - \overset{\textcircled{2}}{4\sqrt{3}}$

$= 2\sqrt{15} - 4\sqrt{3}$

d) $(3-\sqrt{6})(2+\sqrt{24})$ $\quad \textcircled{1}\ \textcircled{2}\ \textcircled{3}\ \textcircled{4}$

$= 6 + 3\sqrt{24} - 2\sqrt{6} - \sqrt{6\cdot 24}$

$= 6 + 3\cdot(\sqrt{4})\cdot(\sqrt{6}) - 2\sqrt{6} - \sqrt{6\cdot 6\cdot 4}$

$= 6 + 6\sqrt{6} - 2\sqrt{6} - (\sqrt{36})\cdot(\sqrt{4})$

$= 6 + 4\sqrt{6} - (\sqrt{6^2})\cdot(\sqrt{2^2})$

Homework: pg 39 #1bdf, 3bdf, 4EOO, 7bdf, 8c, 9abd, 10, 12, 14, 15

$= 6 + 4\sqrt{6} - (6\cdot 2)$

$= -6 + 4\sqrt{6}$

When two expressions are equivalent, for example $2x$ is equal to $\frac{4x}{2}$, the statement $2x = \frac{4x}{2}$ is called an **identity**.

Example 2 Prove the identity below, and state restrictions.

$$Ⓐ \rightarrow \frac{x^2-2x-8}{x^2-x-12} = \frac{x^2-x-6}{x^2-9} \leftarrow Ⓑ$$

Ⓐ $\frac{\cancel{(x-4)}(x+2)}{\cancel{(x-4)}(x+3)}$, $x \neq 4$, $x \neq -3$

$= \frac{(x+2)}{(x+3)}$, $x \neq 4$, $x \neq -3$

Ⓑ $= \frac{\cancel{(x-3)}(x+2)}{(x+3)\cancel{(x-3)}}$, $x \neq 3$, $x \neq -3$

$= \frac{(x+2)}{(x+3)}$, $x \neq 3$, $x \neq -3$

$\therefore$ These two expressions are equal (or same identity) except when $x=3$ and $x=4$

Homework: pg. 83 #C1, C2, C3, 3, 4, 5EOO, 6bdf,8, 9, 15*, 16*