Mr. Park Feb 19 * Tues : Unit Test!

MCR3U

Operations with Radicals (1)

Youtube : "Radical Expressions : Rationalizing the Denominator" by TwinValley Math

Rationalizing the denominator (Change a den. that's irrational to one that's rational)

a) $\frac{6}{\sqrt{2}}$ Multiply radical by itself

b) $\frac{2\sqrt{3}}{3\sqrt{6}}$

* In Math, we do not allow radical number in denominator. So we must learn how to "rationalize" the denominator.

a) $\frac{6}{\sqrt{2}}$ How can you change $\sqrt{2}$ to rational number?

$= \frac{6 \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}} = \frac{6\sqrt{2}}{\sqrt{2^2}} = \frac{6\sqrt{2}}{2}$

$\therefore 3\sqrt{2}$

b) $\frac{2\sqrt{3}}{3\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}}$

$= \frac{2\sqrt{3\times3\times2}}{3 \cdot \sqrt{6^2}} = \frac{2 \cdot \sqrt{3^2} \cdot \sqrt{2}}{3 \cdot 6}$

$= \frac{2 \cdot 3 \cdot \sqrt{2}}{18}$ $\therefore \frac{\sqrt{2}}{3}$

c) $\frac{3}{2-\sqrt{3}}$ Multiply by the conjugate (same numbers, but different sign)

* Conjugate of $(a+b) \rightarrow (a-b)$

$(2-\sqrt{3}) \rightarrow (2+\sqrt{3})$

* Diff Square

$a^2 - b^2 \Rightarrow (a+b)(a-b)$

c) $\frac{3}{(2-\sqrt{3})} \times \frac{(2+\sqrt{3})}{(2+\sqrt{3})} = \frac{(3 \cdot 2) + 3\sqrt{3}}{2^2 - (\sqrt{3})^2} = \frac{6+3\sqrt{3}}{4-3}$

$a = 2, \; b = \sqrt{3}$

$= \frac{6+3\sqrt{3}}{1}$ $\therefore 6 + 3\sqrt{3}$

Simplify

a) $\frac{4 \pm \sqrt{24}}{2}$

b) $\frac{10 \pm \sqrt{125}}{5}$

c) $12 \pm \sqrt{72}$

a) $\dfrac{4 \pm \sqrt{24}}{2} = \dfrac{4 \pm \sqrt{2 \times 3 \times 4}}{2} = \dfrac{4 \pm (\sqrt{6}) \cdot (\sqrt{2^2})}{2}$

$= \dfrac{4 \pm 2\sqrt{6}}{2} = 2 \pm \sqrt{6} = 2+\sqrt{6}$ and $2-\sqrt{6}$

b) $\dfrac{10 \pm \sqrt{125}}{5} = \dfrac{10 \pm \sqrt{5 \times 25}}{5} = \dfrac{10 \pm \sqrt{5^2} \cdot \sqrt{5}}{5}$

$\div 5 \quad \times 5$

$= \dfrac{10 \pm 5\sqrt{5}}{5} = \dfrac{2 \pm \sqrt{5}}{1}$

$\div 5$

$\therefore 2 \pm \sqrt{5}$

c) $12 \pm \sqrt{72} = 12 \pm \sqrt{36 \times 2} = 12 \pm \sqrt{6^2} \cdot \sqrt{2}$

$= 12 \pm 6\sqrt{2}$

HW: Handout #6, 7 (first column), 8, 12, 14, 15, 18, 19

Practise

A

1. Simplify.

a) $2\sqrt{5}+3\sqrt{5}+6\sqrt{5}$

b) $4\sqrt{3}+2\sqrt{3}-\sqrt{3}$

c) $6\sqrt{2}-\sqrt{2}+7\sqrt{2}-3\sqrt{2}$

d) $5\sqrt{7}+3\sqrt{7}-2\sqrt{7}$

e) $8\sqrt{10}-2\sqrt{10}-7\sqrt{10}$

f) $\sqrt{2}-3\sqrt{2}-9\sqrt{2}+11\sqrt{2}$

g) $\sqrt{5}+\sqrt{5}+\sqrt{5}+\sqrt{5}$

2. Simplify.

a) $5\sqrt{3}+2\sqrt{6}+3\sqrt{3}$

b) $8\sqrt{5}-3\sqrt{7}+7\sqrt{7}-4\sqrt{5}$

c) $2\sqrt{2}+3\sqrt{10}+5\sqrt{2}-4\sqrt{10}$

d) $7\sqrt{6}-4\sqrt{13}-\sqrt{13}+\sqrt{6}$

e) $9\sqrt{11}-\sqrt{11}+6\sqrt{14}-3\sqrt{14}-2\sqrt{11}$

f) $12\sqrt{7}+9-3\sqrt{7}+4$

g) $8+7\sqrt{11}-9-9\sqrt{11}$

3. Simplify.

a) $\sqrt{12}+\sqrt{27}$

b) $\sqrt{20}+\sqrt{45}$

c) $\sqrt{18}-\sqrt{8}$

d) $\sqrt{50}+\sqrt{98}-\sqrt{2}$

e) $\sqrt{75}+\sqrt{48}+\sqrt{27}$

f) $\sqrt{54}+\sqrt{24}+\sqrt{72}-\sqrt{32}$

g) $\sqrt{28}-\sqrt{27}+\sqrt{63}+\sqrt{300}$

4. Simplify.

a) $8\sqrt{7}+2\sqrt{28}$ b) $3\sqrt{50}-2\sqrt{32}$

c) $5\sqrt{27}+4\sqrt{48}$ d) $3\sqrt{8}+\sqrt{18}+3\sqrt{2}$

e) $\sqrt{5}+2\sqrt{45}-3\sqrt{20}$

f) $4\sqrt{3}+3\sqrt{20}-2\sqrt{12}+\sqrt{45}$

g) $3\sqrt{48}-4\sqrt{8}+4\sqrt{27}-2\sqrt{72}$

5. Expand and simplify.

a) $\sqrt{2}(\sqrt{10}+4)$

b) $\sqrt{3}(\sqrt{6}-1)$

c) $\sqrt{6}(\sqrt{2}+\sqrt{6})$

d) $2\sqrt{2}(3\sqrt{6}-\sqrt{3})$

e) $\sqrt{2}(\sqrt{3}+4)$

f) $3\sqrt{2}(2\sqrt{6}+\sqrt{10})$

g) $(\sqrt{5}+\sqrt{6})(\sqrt{5}+3\sqrt{6})$

h) $(2\sqrt{3}-1)(3\sqrt{3}+2)$

i) $(4\sqrt{7}-3\sqrt{2})(2\sqrt{7}+5\sqrt{2})$

j) $(3\sqrt{3}+1)^2$

k) $(2\sqrt{2}-\sqrt{5})^2$

l) $(2+\sqrt{3})(2-\sqrt{3})$

m) $(\sqrt{6}-\sqrt{2})(\sqrt{6}+\sqrt{2})$

n) $(2\sqrt{7}+3\sqrt{5})(2\sqrt{7}-3\sqrt{5})$

6. Simplify.

a) $\frac{1}{\sqrt{3}}$ b) $\frac{2}{\sqrt{5}}$ c) $\frac{2}{\sqrt{7}}$

d) $\frac{\sqrt{1}}{\sqrt{2}}$ e) $\frac{5\sqrt{5}}{2\sqrt{3}}$ f) $\frac{2\sqrt{2}}{\sqrt{18}}$

g) $\frac{4\sqrt{2}}{\sqrt{8}}$ h) $\frac{3\sqrt{5}}{\sqrt{3}}$ i) $\frac{4\sqrt{7}}{2\sqrt{14}}$

j) $\frac{3\sqrt{6}}{4\sqrt{10}}$ k) $\frac{7\sqrt{11}}{2\sqrt{3}}$ l) $\frac{2\sqrt{5}}{5\sqrt{2}}$

7. Simplify.

a) $\frac{1}{\sqrt{2}+2}$ b) $\frac{3}{\sqrt{5}-1}$

c) $\frac{\sqrt{2}}{\sqrt{6}-3}$ d) $\frac{2}{\sqrt{6}+\sqrt{3}}$

e) $\frac{3}{\sqrt{5}-\sqrt{2}}$ f) $\frac{\sqrt{3}}{\sqrt{3}+\sqrt{2}}$

g) $\frac{2\sqrt{6}}{2\sqrt{6}+1}$ h) $\frac{\sqrt{2}-1}{\sqrt{2}+1}$

i) $\frac{\sqrt{2}+\sqrt{5}}{\sqrt{6}-\sqrt{10}}$ j) $\frac{2\sqrt{7}+\sqrt{5}}{3\sqrt{7}-2\sqrt{5}}$

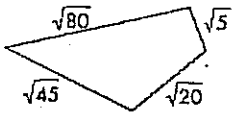
Solutions:

Section 2.4, pp. 139–142

1. a) $11\sqrt{5}$ b) $5\sqrt{3}$ c) $9\sqrt{2}$ d) $6\sqrt{7}$ e) $-\sqrt{10}$ f) 0 g) $4\sqrt{5}$
2. a) $8\sqrt{3}+2\sqrt{6}$ b) $4\sqrt{5}+4\sqrt{7}$ c) $7\sqrt{2}-\sqrt{10}$ d) $8\sqrt{6}-5\sqrt{13}$
e) $6\sqrt{11}+3\sqrt{14}$ f) $13+9\sqrt{7}$ g) $-1-2\sqrt{11}$ **3.** a) $5\sqrt{3}$ b) $5\sqrt{5}$
c) $\sqrt{2}$ d) $11\sqrt{2}$ e) $12\sqrt{3}$ f) $5\sqrt{6}+2\sqrt{2}$ g) $5\sqrt{7}+7\sqrt{3}$ **4.** a) $12\sqrt{7}$
b) $7\sqrt{2}$ c) $31\sqrt{3}$ d) $12\sqrt{2}$ e) $\sqrt{5}$ f) $9\sqrt{5}$ g) $24\sqrt{3}-20\sqrt{2}$
5. a) $2\sqrt{5}+4\sqrt{2}$ b) $3\sqrt{2}-\sqrt{3}$ c) $2\sqrt{3}+6$ d) $12\sqrt{3}-2\sqrt{6}$
e) $\sqrt{6}+4\sqrt{2}$ f) $12\sqrt{3}+6\sqrt{5}$ g) $23+4\sqrt{30}$ h) $16+\sqrt{3}$
i) $26+14\sqrt{14}$ j) $28+6\sqrt{3}$ k) $13-4\sqrt{10}$ l) 1 m) 4 n) -17
6. a) $\frac{\sqrt{3}}{3}$ b) $\frac{2\sqrt{5}}{5}$ c) $\frac{2\sqrt{7}}{7}$ d) $\frac{\sqrt{2}}{2}$ e) $\frac{5\sqrt{15}}{6}$ f) $\frac{2}{3}$ g) 2 h) $\sqrt{15}$ i) $\sqrt{2}$
j) $\frac{3\sqrt{15}}{20}$ k) $\frac{7\sqrt{33}}{6}$ l) $\frac{\sqrt{10}}{5}$ **7.** a) $\frac{2-\sqrt{2}}{2}$ b) $\frac{3+3\sqrt{5}}{4}$
c) $-\frac{3\sqrt{2}+2\sqrt{3}}{3}$ d) $\frac{2\sqrt{6}-2\sqrt{3}}{3}$ e) $\sqrt{5}+\sqrt{2}$ f) $3-\sqrt{6}$
g) $\frac{24-2\sqrt{6}}{23}$ h) $3-2\sqrt{2}$ i) $-\frac{5\sqrt{2}+2\sqrt{3}+2\sqrt{5}+\sqrt{30}}{4}$
j) $\frac{52+7\sqrt{35}}{43}$ **8.** $10\sqrt{5}$ **9.** $(\sqrt{3}+1)^2$, $\sqrt{3}(\sqrt{3}+1)$, $(\sqrt{3}+1)(\sqrt{3}-1)$, $(1-\sqrt{3})^2$ **10.** a) $6\sqrt{8}+\sqrt{8}-5\sqrt{8}$ b) It is twice as large as the others. **11.** 1 **12.** a) $8\sqrt{6}-6$
b) $8\sqrt{2}+2\sqrt{3}$ **13.** $13-4\sqrt{10}$ **14.** $38\sqrt{15}-38\sqrt{2}$
15. $2\sqrt{7}+2\sqrt{5}$ **16.** a) $x^2-6x+7=0$ b) $x^2+2x-11=0$
c) $4x^2-8x-9=0$ **17.** a) $5\sqrt[3]{2}$ b) $5\sqrt[3]{3}$ c) $19\sqrt[3]{4}$ d) $13\sqrt[3]{2}$
e) $-\sqrt[3]{2}$ f) $\sqrt[3]{4}$ g) $3\sqrt[3]{5}$ h) $4\sqrt[3]{6}$ **18.** $97+56\sqrt{3}$ **19.** a) $8+4\sqrt{13}$
b) $3\sqrt{2}+2\sqrt{5}$ **20.** The statement is sometimes true. It is true if and only if one or both of a and b are equal to zero.

Apply, Solve, Communicate

8. Measurement Express the perimeter of the quadrilateral in simplest radical form.



B

9. Without using a calculator, arrange the following expressions in order from greatest to least.

$\sqrt{3}(\sqrt{3}+1), (\sqrt{3}+1)(\sqrt{3}-1), (1-\sqrt{3})^2, (\sqrt{3}+1)^2$

10. a) Without using a calculator, decide which of the following radical expressions does not equal any of the others.

$\frac{60}{\sqrt{450}}$ $\quad 6\sqrt{2}-4\sqrt{2}$ $\quad \frac{4}{\sqrt{2}}$ $\quad 6\sqrt{8}+\sqrt{8}-5\sqrt{8}$ $\quad \frac{8}{\sqrt{18}}+\frac{4}{\sqrt{18}}$

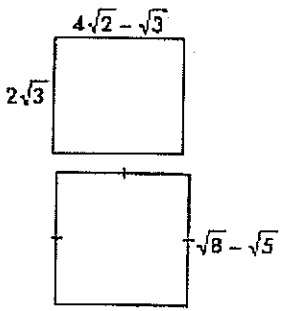
b) Communication How is the radical expression you identified in part a) related to each of the others?

11. Nature Many aspects of nature, including the number of pairs of rabbits in a family and the number of branches on a tree, can be described using the Fibonacci sequence. This sequence is 1, 1, 2, 3, 5, 8, ...
The expression for the nth term of the Fibonacci sequence is called Binet's formula. The formula is

$$F_n = \frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^n - \frac{1}{\sqrt{5}}\left(\frac{1-\sqrt{5}}{2}\right)^n.$$

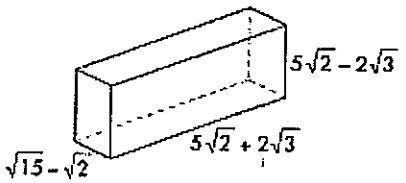
Use Binet's formula to find F_2.

12. Measurement Write and simplify an expression for
a) the area of the rectangle
b) the perimeter of the rectangle



13. Measurement Write and simplify an expression for the area of the square.

14. Measurement Express the volume of the rectangular prism in simplest radical form.



15. Application If a rectangle has an area of 4 square units and a width of $\sqrt{7}-\sqrt{5}$ units, what is its length, in simplest radical form?

16. Inquiry/Problem Solving Write a quadratic equation in the form $ax^2+bx+c=0$ with the given roots.

a) $3+\sqrt{2}$ and $3-\sqrt{2}$ **b)** $-1+2\sqrt{3}$ and $-1-2\sqrt{3}$

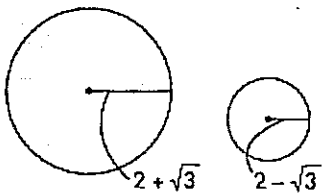
c) $1+\frac{\sqrt{13}}{2}$ and $1-\frac{\sqrt{13}}{2}$

C

17. Simplify.

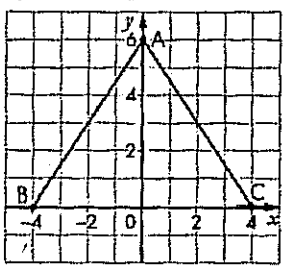
a) $\sqrt[3]{16}+\sqrt[3]{54}$ **b)** $\sqrt[3]{24}+\sqrt[3]{81}$
c) $2(\sqrt[3]{32})+5(\sqrt[3]{108})$ **d)** $\sqrt[3]{54}+5(\sqrt[3]{16})$
e) $\sqrt[3]{16}-\sqrt[3]{54}$ **f)** $\sqrt[3]{108}-\sqrt[3]{32}$
g) $2(\sqrt[3]{40})-\sqrt[3]{5}$ **h)** $5(\sqrt[3]{48})-2(\sqrt[3]{162})$

18. Measurement Express the ratio of the area of the larger circle to the area of the smaller circle in simplest radical form.

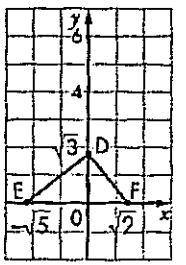


19. Coordinate geometry State the perimeter of each of the following triangles in simplest radical form.

a)



b)



20. Equation Is the statement $\sqrt{a+b}=\sqrt{a}+\sqrt{b}$ always true, sometimes true, or never true? Explain.