March 6 Park

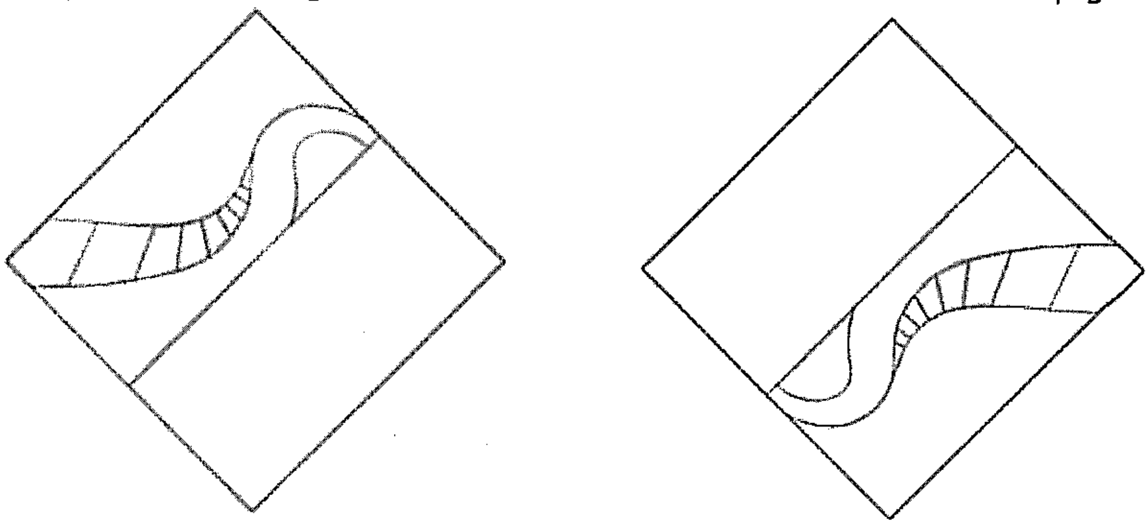
Youtube: "Inverse Functions – The Basics" by PatrickJMT

MCR3U

INVERSES

Minds on

Complete the drawings as if the line was a mirror line. Do not turn the page!



Definition: The inverse of a function is the "reverse" of the original function.

Example: The inverse of $y = x^2$ is $y = \pm\sqrt{x}$

The inverse may or may not be a function. If the inverse is a function, and the original function is f, then f^{-1} is the name of the inverse. $f^{-1}(x)$

***Be careful! f^{-1} is read "f inverse" and the -1 is NOT an exponent! It does not mean 1/f.**

In a table of values, the x and y values are switched for the inverse function.
If $(a, b) \in f$, then $(b, a) \in f^{-1}$.

$y = x^2$

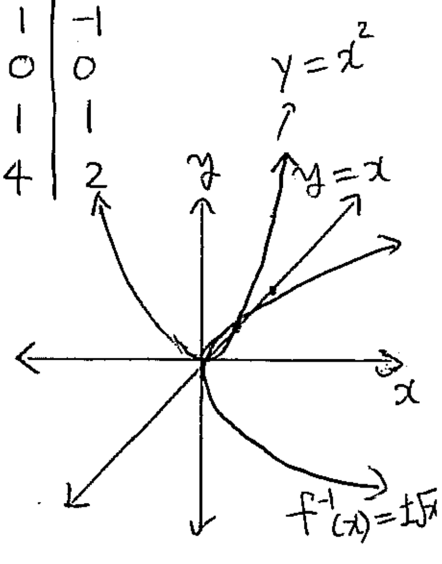
x	y
-2	4
-1	1
0	0
2	4

$y = \pm\sqrt{x}$

x	y
4	-2
1	-1
0	0
1	1
4	2

* The graph of an inverse is the reflection of f in the y = x line.

* Algebraically, you switch the x and y and then you re-solve it for y.



Example 1

a) Find the inverse of f={(0, 3), (1, 3), (2, 3), (3, 3), (4, 3), (5, 3)}.

$f^{-1} = \{(3,0), (3,1), (3,2), (3,3), (3,4), (3,5)\}$

b) **Invariant points** are points that do not change during a transformation. What point(s) are invariant?

(3,3) Any points which lie on $y=x$ line

c) Is the inverse a function? Explain your answer.

No one x value has multiple y-values

d) State the domain and range of f.

$D = \{0, 1, 2, 3, 4, 5\}$ $R = \{3\}$

e) State the domain and range of the inverse.

$D = \{3\}$ $R = \{0, 1, 2, 3, 4, 5\}$

Example 2 Questions that use Conventional Variables

a) Find the inverse of the linear function
$f(x) = 3x + 2$.

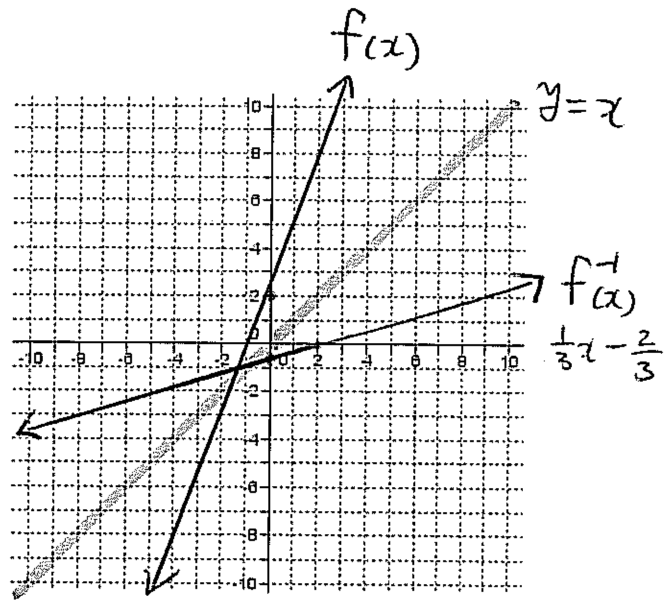
$y = 3x + 2$ Step 1: change $f(x)$ to y

$x = 3y + 2$ Step 2: switch x and y

$x - 2 = 3y$ Step 3: Rearrange to get

$\div 3 \quad \div 3$ y (= Isolate y)

$y = \frac{x-2}{3}$ or $f^{-1}(x) = \frac{1}{3}x - \frac{2}{3}$



b) Is the inverse a function?

Yes it passes vertical line test.

c) Graph the function and its inverse.

***Note: When you have a word problem and the variables are specified, you do NOT switch the variables.**

Example 3 Application Question

The equation used for converting degrees Celsius to degrees Fahrenheit is $F = \frac{9}{5}C + 32$. (In function notation this would be $F(C) = \frac{9}{5}C + 32$.)

a) Find the inverse of this function.

$$F = \frac{9}{5}C + 32$$

rearrange it so you can isolate C by itself.

$$F - 32 = \frac{9}{5}C$$

$$\times \frac{5}{9} \quad \times \frac{5}{9} \quad \times \frac{5}{9}$$

$$\frac{5}{9}F - \frac{160}{9} = C$$

$$C = \frac{5}{9}F - \frac{160}{9}$$

$$\therefore F^{-1} = \frac{5F}{9} - \frac{160}{9}$$

b) When would the inverse be useful?

It can be used to convert Fahrenheit to Celcius.

Example 4 Restricting the domain to make the inverse a function

a) Find the inverse of $h(x) = x^2 + 5$ algebraically.

$$y = x^2 + 5$$

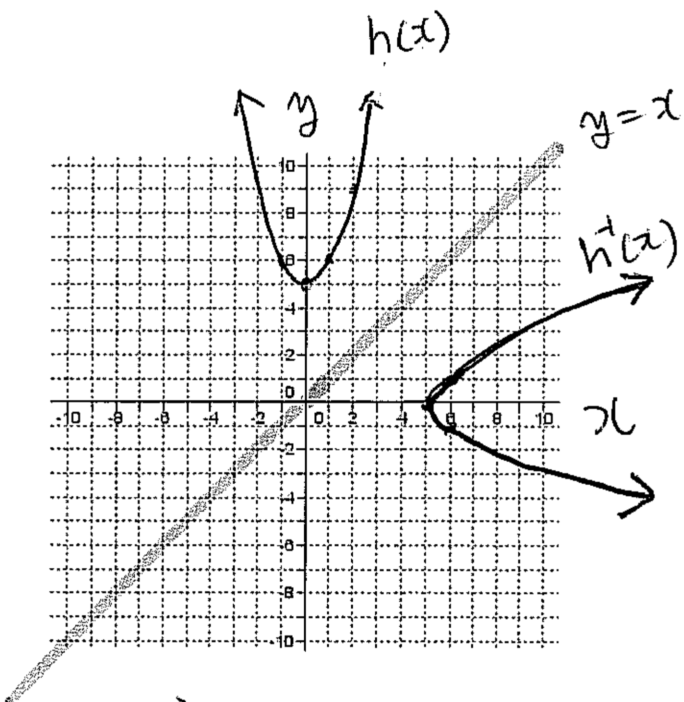
$$x = y^2 + 5$$

$$\pm\sqrt{x-5} = \sqrt{y^2}$$

$$\pm\sqrt{x-5} = y$$

$$\therefore h^{-1}(x) = \pm\sqrt{x-5}$$

b) Graph the function and it's inverse.



c) Is the inverse a function?

No it is not a function.

d) Restrict the domain of $h(x)$ to make the inverse a function.

We can restrict the domain (of h(x)) to $\{x \in \mathbb{R},\ x \geq 0\}$ or $\{x \in \mathbb{R},\ x \leq 0\}$

17ab

Homework: pg. 138 #1ab, 3, 4bd, 5, 7abc, 11, ~~12~~, 15ii, iii, iv, ~~17ab~~