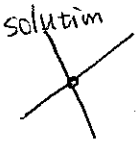
MPM2D

Solving Linear Systems by Graphing

Linear Equation → e.g. $y = mx + b$ – an equation that relates two variables so that ordered pairs satisfying the equation form a straight line on a graph

Linear System – two or more lines that are considered at the same time

How many ways can two lines meet?

① Two lines meet once →  one solution

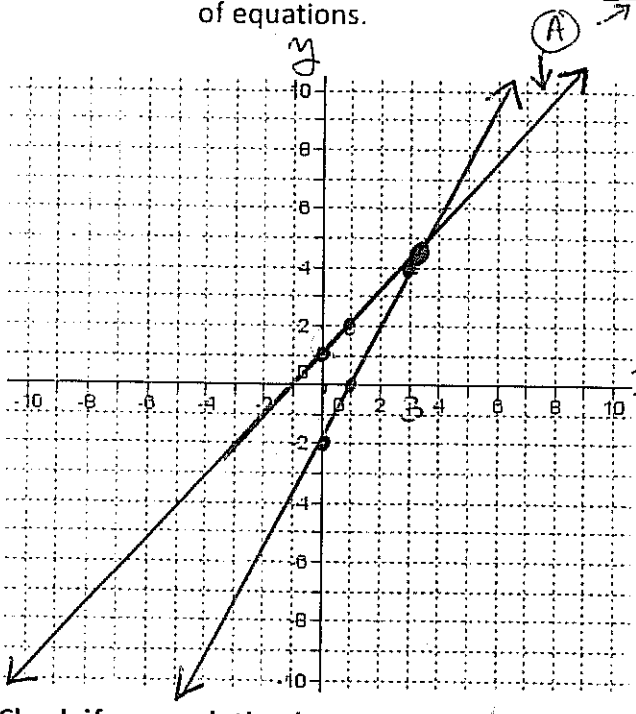
② // // never meet →  (= parallel) → no solution

③ // // meet everywhere →  (= same slope and y intercept)
↳ infinite # of solution

When we say "**solve**" a linear system, we mean find the point of intersection

POI = intersection point, the point where two lines cross, or the point that satisfies both equations.

Example 1 The equations for two lines are $x - y = -1$ and $2x - y = 2$. Solve the system of equations.



Ⓐ $x - y = -1$ (0, 1)

$-y = -1 - x$
$\times(-1) \quad \times(-1) \quad \times(-1)$

$y = x + 1$ → y int = 1

slope $= \frac{1}{1} = \frac{\Delta y}{\Delta x} = \frac{\text{rise}}{\text{run}}$

Ⓑ $2x - y = 2$ (0, −2)

$-y = 2 - 2x$
$\times(-1) \quad \times(-1) \quad \times(-1)$

$y = 2x - 2$ → y int = −2

slope $= 2 = \frac{2}{1} = \frac{\Delta y}{\Delta x} = \frac{\text{rise}}{\text{run}}$

Check if your solution is correct: (3, 4) → when $x = 3$, $y = 4$

Ⓐ Substitute (3, 4)

$3 - 4 = -1$

$-1 = -1$

∴ LS = RS

Ⓑ Substitute (3, 4)

$2(3) - (4) = 2$

$6 - 4 = 2$

$2 = 2$

∴ LS = RS

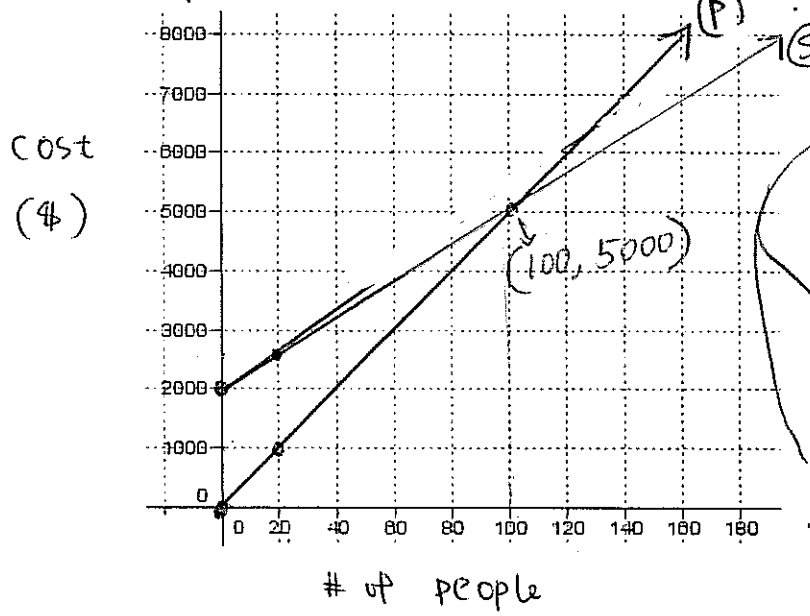
Example 2 Brian and Catherine want to rent a banquet hall for their wedding. Party Hall charges $50 per person. Super Cheap Hall charges a fixed rate of $2000, plus $30 per person.

Let x be number of people

Let Y be cost

a) When is the cost the same?

Y = Dependent variable



* Party Hall → y int = 0

$Y = 50x$ → (P)

$\frac{50}{1} = \frac{\text{rise}}{\text{run}}$

* Super Cheap

$Y = 30x + 2000$ → (S)

$\frac{30}{1} \times \frac{20}{20} = \frac{600}{20} = \frac{\text{rise}}{\text{run}}$

$* \frac{50}{1} \times \frac{20}{20} = \frac{1000}{20} = \frac{\text{rise}}{\text{run}}$

x = Independent variable

b) If Brian and Catherine expect to have 80 people, which hall is the better option for them?

Party Hall

c) Under what conditions would you recommend Party Hall? Under what conditions would you recommend Super Cheap Hall?

If less than 100 people ~~will~~ would show up, then we should use Party Hall.

If you know that more than 100 people would show up, then you use Super cheap.

Example 3

Graph the lines $y = 3x + 4$ and (a)
$2y - 6x = -10$. → (b)

(a) $y = 3x + 4$ $\quad y$ int $(0, 4)$

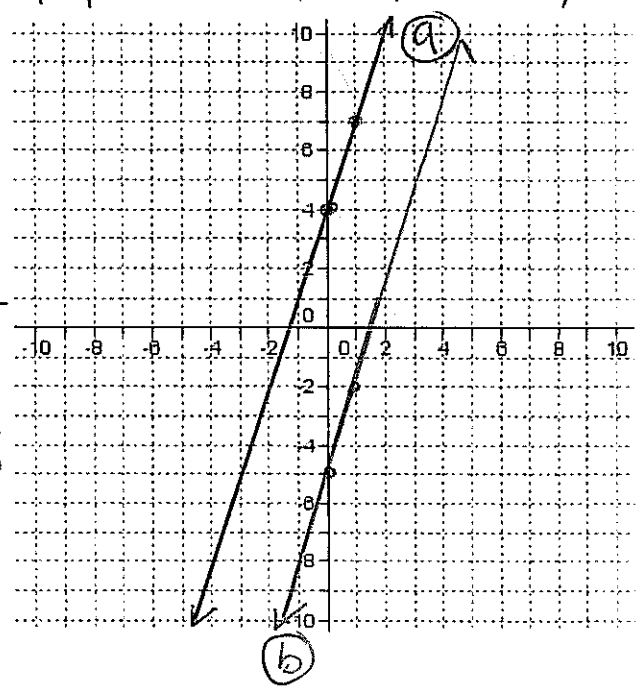
$m = \frac{3}{1} = \frac{\text{rise}}{\text{run}}$

(b) $\frac{2y}{2} = \frac{-10}{2} + \frac{6x}{2}$ $\quad \therefore y$ int $(0, -5)$

$y = -5 + 3x$ $\quad \therefore m = \quad = \frac{\text{rise}}{\text{run}}$

What is the POI?

None because they are parallel to each other.



Ⓑ y int: $4y = 8$

$y = 2$ (A)

→ x int: $2x + 0 = 8$

$2x = 8$

$x = 4$ (4,0)

Example 4

1. On the same grid graph the lines $x + 2y = 4$ and $2x + 4y = 8$ using x- and y-intercepts.

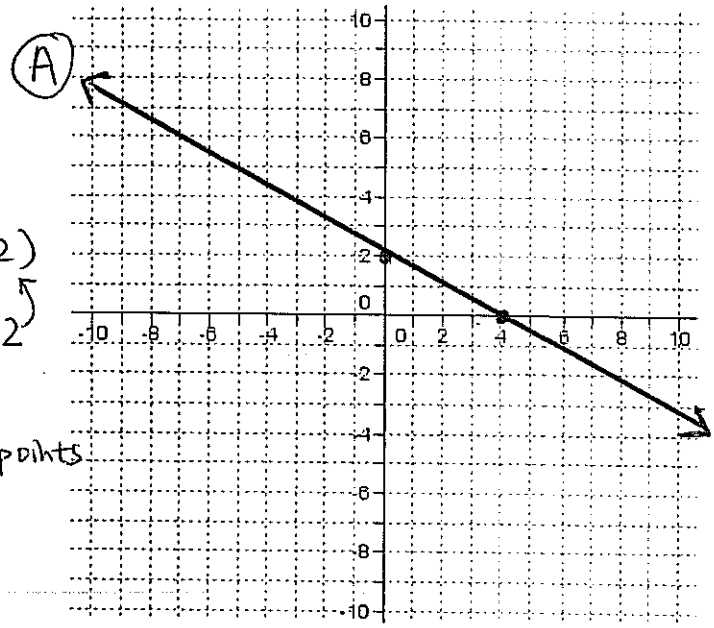
Ⓐ x int: sub $y = 0$ → Ⓐ

$x + (2 \cdot 0) = 4$

$x = 4 \rightarrow (4, 0)$

y int: sub $x = 0$ → Ⓐ (0, 2)

$0 + 2y = 4 \rightarrow 2y = 4 \rightarrow y = 2$



a) What is the POI?

everywhere or infinite # of points

b) How are the equations related?

They are the same.

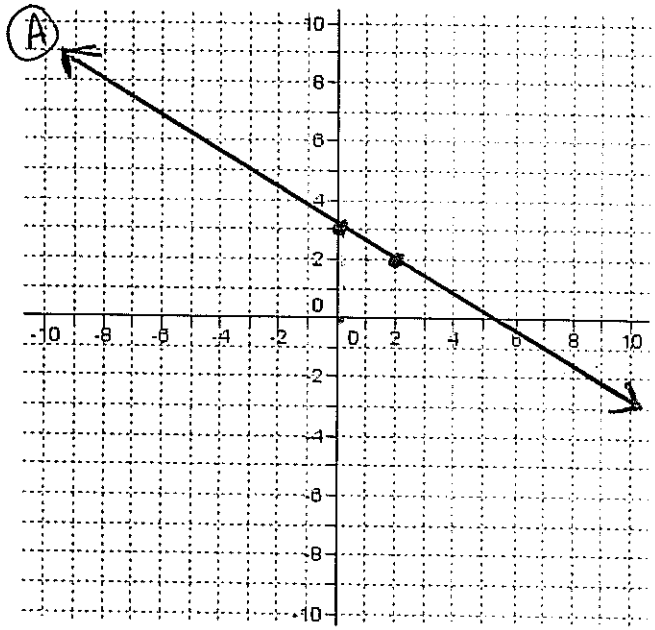
2. On the same grid graph the lines $y = -\frac{1}{2}x + 3$ and $x + 2y = 6$.

Ⓐ $y = -\frac{1}{2}x + 3$ y int (0, 3)

$m = \frac{-1}{2} = \frac{\text{rise}}{\text{run}}$

Ⓑ $\frac{2y}{2} = \frac{6}{2} - \frac{x}{2}$ y int (0, 3)

$y = -\frac{x}{2} + 3$ → $m = \frac{-1}{2}$



a) How are the graphs related?

They are the same.

b) How are the equations related?

Same.

The graphs in example 4 are called equivalent linear equations.

HW

1. a) Without graphing, figure out which two of the following are equivalent linear equations.

$y - x + 5 = 0$ $y = 3x + 15$ $2y = 2x - 10$

2. Which one of the following is not equivalent to the others?

$2x - 4y = 8$ $y = \frac{1}{2}x - 2$ $2y - x - 4 = 0$

3. Write an equivalent relation for each of the following:

a) $2x + 2y = 12$ b) $x + y = 4$ c) $y = 2/3x + 1$

Equivalent Linear Systems

Compare the following linear systems:

Linear System 1	**Linear System 2**
$y = x - 1$ $y = -\frac{1}{2}x + 2$	$2x - 2y - 2 = 0$ $2y + x = 4$

How do you think the POIs of the systems would compare? Why?

Homework: pg. 60 #(1-4), 5, 7-9 The ones in the bracket are optional
Pg. 70 #1-4, 5EOO, 6, 16 **Thinking Practice:** 17, 21-23