

Recall: What does "solving" a linear system mean?

= Find POI (= point of Intersection)

If the volume of a sphere is given by the equation  $V = \frac{3}{4}\pi r^3$ , find the volume of a sphere with 2 cm.  $r=2 \rightarrow$  substitute into  $\nearrow$

$$V = \frac{3}{4}\pi (2)^3 = \frac{3\pi \cdot 8}{4} = \frac{24\pi}{4} = 6\pi = 18.85 \text{ cm}$$

Graphing doesn't always give an accurate answer, and it can take a long time to graph.

Substitution

– an algebraic method of solving a linear system by

substituting

one variable from one equation into the other equation.

**Example 1** Solve Using the Method of Substitution

The lines  $y = -x + 8$  and  $x - y = 4$  intersect at right angles. Find the coordinates of the point of intersection.

$$y = -x + 8 \rightarrow \textcircled{a}$$

$$x - y = 4 \rightarrow \textcircled{b}$$

(Step 1) sub  $y = -x + 8$  into  $\textcircled{b}$

$$\textcircled{b} \quad x - (-x + 8) = 4$$

$$x + x - 8 = 4$$

$$2x - 8 = 4$$

$$2x = 4 + 8$$

$$\frac{2x}{2} = \frac{12}{2}$$

$$\therefore x = 6$$

(Step 2): sub  $x = 6$  into  $\textcircled{a}$  (or  $\textcircled{b}$ )

$$\textcircled{a} \quad y = -(6) + 8$$

$$\therefore y = 2$$

$$\therefore \text{POI } (6, 2)$$

**Example 2** Solve Using the Method of Substitution

Find the solution to the linear system

$$x + y = 5 \rightarrow \textcircled{a}$$

$$3x - y = 7 \rightarrow \textcircled{b}$$

Step 1: Rearrange  $\textcircled{a}$

$$\textcircled{a} \quad \begin{array}{r} x + y = 5 \\ -x \quad -x \end{array}$$

$$\textcircled{a}' \quad y = 5 - x$$

sub  $\textcircled{a}'$  into  $\textcircled{b}$

$$\textcircled{b} \quad 3x - y = 7$$

$$3x - (5 - x) = 7$$

$$3x - 5 + x = 7$$

$$4x - 5 = 7 \quad \begin{array}{c} +5 \quad +5 \end{array}$$

$$\frac{4x}{4} = \frac{12}{4}$$

$$\therefore x = 3$$

Step 2: sub  $x = 3 \rightarrow \textcircled{a}$

$$\textcircled{a} \quad 3 + y = 5$$

$$y = 5 - 3 = 2 \quad \therefore y = 2$$

$$\therefore \text{POI } (3, 2)$$

Step 1

Example 3 Solve Using the Method of Substitution

Where do the lines  $2x - y = 4$  and  $4x + y = 9$  intersect?

Rearrange

(A)

(B)

✓ (B)  $4x + y = 9$

$$y = 9 - 4x$$

sub  $y = 9 - 4x$  into (A)

(A)  $2x - (9 - 4x) = 4$

$$2x - 9 + 4x = 4$$

$$6x - 9 = 4$$

$$6x = 4 + 9$$

$$\frac{6x}{6} = \frac{13}{6} \rightarrow x = \frac{13}{6}$$

Step 2: sub  $x = \frac{13}{6}$  into (B)

(B)  $4\left(\frac{13}{6}\right) + y = 9$

$$\frac{52}{6} + y = 9$$

$$y = 9 - \frac{52}{6}$$

$$y = \frac{54 - 52}{6}$$

$$y = \frac{2 \div 2}{6 \div 2} = \frac{1}{3}$$

∴ POI  $\left(\frac{13}{6}, \frac{1}{3}\right)$

Example 4 Stephanie has five more fish in her aquarium than Brett has. The two have a total of 31 fish. How many fish does Stephanie have? How many fish does Brett have?

$$S = B + 5 \rightarrow (A)$$

$$B + S = 31 \rightarrow (B)$$

sub  $S = B + 5$  into (B)

(B)  $B + (B + 5) = 31$

$$2B + 5 = 31$$

$$2B = 31 - 5$$

$$\frac{2B}{2} = \frac{26}{2}$$

$$\therefore B = 13$$

$S = \#$  of fish Stephanie has

$B = \#$  // Brett has

Step 2: sub  $B = 13$  into (A)

(A)  $S = 13 + 5$

$$S = 18$$

∴ Stephanie has 18 fish and  
Brett has 13 fish.