MPM2D

Linear Systems - Elimination

Feb 23

* Quiz on Wed, Feb 25

There are 3 different ways to solve linear systems.

1) Graphing 2 lines and look at POI
2) Substitution
3) Elimination

The method of Elimination is easier than substitution if we cannot easily isolate either variable. For example in the equation $5y + 3x = 17$, neither x nor y is easy to isolate.

Definition:

Coefficient - The number by which a variable is multiplied e.g. $10x$

Example 1 Solve a linear system using the method of elimination

Solve the system of linear equations

ⓐ $3x + y = 19$

ⓑ + $4x - y = 2$

ⓐ + ⓑ: $7x + 0y = 21$

$\frac{7x}{7} = \frac{21}{7}$

$x = 3$

ⓐ $3x + y = 19$

$3(3) + y = 19$

$9 + y = 19$

$-9 \qquad -9$

$y = 10$

Step 1: Add (or subtract or multiply) two equations in order to eliminate one of two variables.

Step 2: Solve the remaining equation for x or y.

Step 3: Substitute the answer from step 2 into either ⓐ or ⓑ

Step 4: Solve the remaining equation for x or y.

Step 5: State ∴ statement

∴ (3, 10) is POI

Check your solution

Ⓐ $3x + y = 19$

LS	RS
$3(3)+10$	19
$9+10$	
19	∴ LS = RS

Ⓑ $4x - y = 2$

LS	RS
$4(3) - 10$	2
$12 - 10$	
2	2

∴ LS = RS

You can **either add or subtract** the equations in order to eliminate one variable.

Example 2 Determine whether we should add or subtract the equations. **Justify** your choice.

a) $3x + 2y = 5$
$x - 2y = -1$

Add because y terms' coefficients have opposite signs.

b) $x + y = 3$
$2x + y = 3$

Subtract because y term has same coefficient.

c) $2x + 5y = 3$
$2x - y = -3$

Subtract

d) $2x + y = 7$
$x - y = -1$

Add

By looking at the coefficient of x or y term, you can determine

How do we know that we can just add or subtract the equations? ~~**How do we know the intersection with the new equation will still work?**~~

Graph the linear system and find the point of intersection.

(A) $x + 2y = 4$
(B) $x - y = 1$

(A) $\frac{2y}{2} = \frac{4}{2} - \frac{x}{2}$

$y = 2 - \frac{x}{2}$

$\therefore$ y int $= 2$ $\quad m = -\frac{1}{2} = \frac{\text{rise}}{\text{run}}$

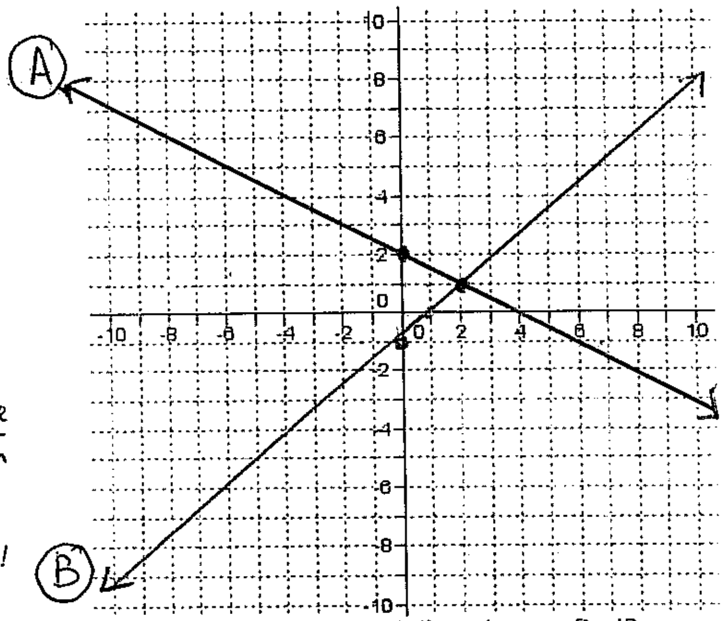
(B) $x - y = 1$

$\frac{-y}{x-1} = \frac{1}{x-1} - \frac{x}{x-1}$

$y = x - 1$

$\therefore$ y int $= -1$

$m = \frac{1}{1} = \frac{\text{rise}}{\text{run}}$



**Don't forget to label the lines on your graph!*

~~**Add the left and the right sides of the equations.** Graph the new equation. What do you find?~~

$\therefore$ POI $= (2, 1)$

~~**Subtract the left and the right sides of the equations.** Graph the new equation. What do you find?~~

What if the coefficient of x and y are not the same? You multiply (or divide) one of the two equations so that coefficients of x (or y) will be same number.

Example 3 Solve Using Elimination

Solve the linear system

(A) $10x + 4y = -1$

(B) $8x - 2y = 7$

(B)x2 → $16x - 4y = 14$

(A) $+ \quad 10x + 4y = -1$

(A) + 2(B) $26x + 0 = 13$

$\frac{26x}{26} = \frac{13}{26}$

$x = \frac{13 \div 13}{26 \div 13} = \frac{1}{2}$

$\therefore x = \frac{1}{2}$

Sub $x = \frac{1}{2}$ → (A)

$10\left(\frac{1}{2}\right) + 4y = -1$

$5 + 4y = -1$

$4y = -1 - 5$

$4y = -6$

$\div 4 \qquad \div 4$

$y = \frac{-6 \div 2}{4 \div 2} = \frac{-3}{2}$

$\therefore$ POI $= \left(\frac{1}{2}, \frac{-3}{2}\right)$

Example 3 Find a Point of Intersection Using Elimination

Find the point of intersection of the linear system.

(A) $4x + 3y = 13$

(B) $5x - 4y = -7$

(A) x 4 → $16x + 12y = 52$

(B) x 3 → $15x - 12y = -21$

4(A) + 3(B) $\frac{31x}{31} = \frac{31}{31}$

$x = 1$

sub $x=1$ → (A)

(A) $4 \cdot 1 + 3y = 13$

$4 + 3y = 13$

$3y = 13 - 4$

$y = 9 \div 3 = 3$ $\therefore$ POI $= (1, 3)$

Example 4 Solve a Problem Using the Method of Elimination

A small store sells used CDs and DVDs. The CDs sell for $9 each. The DVDs sell for $11 each. Cody is working part time and sells a total of $204 worth of CDs and DVDs during his shift. He knows that 20 items were sold. He needs to tell the store owner how many of each type were sold. How many CDs did Cody sell? How many DVDs?

Let C = # of CD sold

Let D = # of DVD sold

$C + D = 20$ (A)

$9C + 11D = 204$ (B)

(A) $\times 9 \rightarrow 9C + 9D = 180$

(B) $- \quad 9C + 11D = 204$

(A) − (B) $\quad 0 + -2D = -24$

$\div -2 \quad \div -2$

$D = 12$

Sub $D = 12$ into (A)

$C + 12 = 20$

$C = 20 - 12$

$C = 8$

∴ They sold 12 DVDs and 8 CDs.

Substitution or Elimination? Which one is easier? Justify your choice.

a) $x + y = 7$ (A)
$x = y + 3$ (B)

Substitution because (B) is nicely organized into $x = ?$ format.

b) $4x + 3y = -1.9$ (A)
$2x - 7y = 3.3$ (B)

Elimination because I can multiply by 2 to equation (B) then both x terms contain 4 as coefficient.

c) $5x - 4y + 13 = 0$ (A)
$7x - y + 9 = 0$ (B)

Elimination because I can multiply (B) by 4, then both y terms contain −4 as coefficient.

d) $4k + 5h = -5$ (A)
$3k + 7h = 6$ (B)

Substitution because I can rearrange (B) to isolate k. Then I can sub k into (A)

Homework: pg. ~~39~~ 101 **# 1-3, 6, 7**

Bolded questions are mandatory.