

MCR3U  
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Combinations of Transformations

**Stretched vertically by a factor of  $a$**

- If  $a > 1$ , the graph is expanded
- If  $0 < a < 1$  the graph is compressed
- If  $a < 0$ , the graph is reflected in the  $x$ -axis

= reflection

**Translated vertically  $c$  units**

- If  $c > 0$ , the graph shifts up
- If  $c < 0$ , the graph shifts down

$$y = af(k(x-d)) + c$$

**Stretched horizontally by a factor of  $1/k$**

- If  $k > 1$ , the graph is compressed
- If  $0 < k < 1$  the graph is expanded
- If  $k < 0$ , the graph is reflected in the  $y$ -axis

= reflection

**Translated horizontally  $d$  units**

- If the sign is negative, the graph shifts to the right
- If the sign is positive, the graph shifts to the left

\* If the equation is not in this form, you have to rearrange or simplify so the equation looks like  $y = af(k(x-d)) + c$  or factor

The  $a$  affects the graph  $y = f(x)$  by stretching or compressing vertically by a factor of  $a$ .

If the  $a$  is negative, there is a reflection about the  $x$  axis.

The  $d$  affects the graph  $y = f(x)$  by translating horizontally  $d$  units.

The  $c$  affects the graph  $y = f(x)$  by translating vertically  $c$  units.

The  $k$  affects the graph  $y = f(x)$  by stretching or compressing horizontally by

a factor of  $\left(\frac{1}{k}\right)$  = only  $x$  coordinates will be multiplied by  $\frac{1}{k}$  for given points.

If the  $k$  is negative, there is a reflection about the  $y$  axis.

Does the order of transformations matter? Shifting always occur last.

An easy way is to transform left to right on the equation.

F: Inverse Function

M: Review for Unit Test

Wed: // // // //

TH: Unit Test

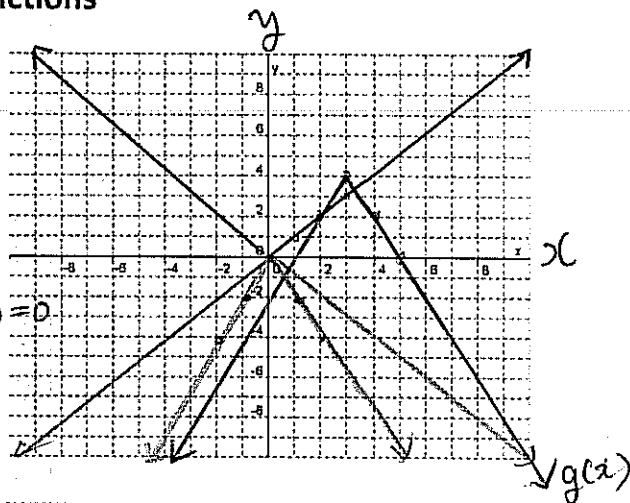
### Graphing Point by Point (alternative method of graphing)

- 1) Start by graphing the base function ( $y = x^2, y = \sqrt{x}, y = x^3, y = |x|, y = \frac{1}{x}$ ).
- 2) Graph  $a$  by taking the significant point  $(x, y)$  and multiply only the  $y$ -values by  $a$  so that the point  $(x, y)$  becomes  $(x, ay)$ .
- 3) Graph  $k$  by taking the significant point  $(x, y)$  and multiply only the  $x$ -values by  $\frac{1}{k}$  so that the point  $(x, y)$  becomes  $(\frac{x}{k}, y)$ .
- 4) Graph  $d$  by taking the significant point  $(x, y)$  and adding  $d$  to the  $x$ -value so that the point  $(x, y)$  becomes  $(x + d, y)$ .
- 5) Graph  $c$  by taking the significant point  $(x, y)$  and adding  $c$  to the  $y$ -value so that the point  $(x, y)$  becomes  $(x, y + c)$ .

### Example 1 – Sketching Graphs of Transformed Functions

1. Given the graph of  $f(x) = |x|$ , describe how you would graph  $g(x) = -2f(x - 3) + 4$  using transformations.

- ① Draw parent function  $f(x) = |x|$
- ② reflect it in  $x$  axis.
- ③ stretch vertically by 2
- ④ shift 3 units to the right = what makes  $() = 0$
- ⑤ shift up 4 units.



2. a) Using transformations, sketch the graph of

$$y = \sqrt{2x + 4} + 1.$$

Parent F:  $f(x) = \sqrt{x}$

Hint: Rewrite  $2x + 4$  in factored form to determine the horizontal translation.

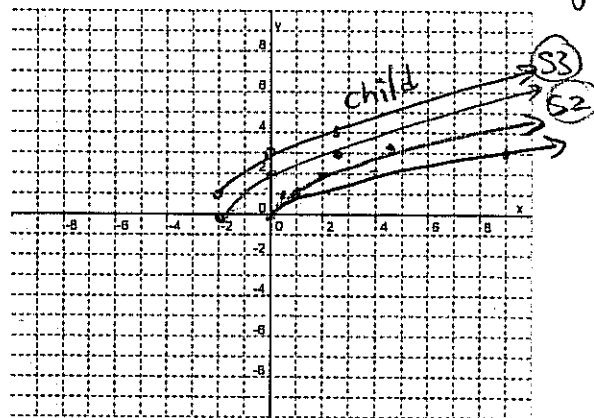
$$y = \sqrt{2(x+2)} + 1$$

① Draw parent function

② Horizontal compression by  $\frac{1}{2}$

③ shift left 2 units

④ shift up 1 unit.



$x, y$	$x, y$
1, 1	$\frac{1}{2}, 1$
4, 2	2, 2
9, 3	$\frac{9}{2}, 3$