

March 4

**Example 2 - Writing Equations of Transformed Functions**

1. The function  $y = f(x)$  has been transformed into  $y = af(k(x - d)) + c$ . Write the following in the appropriate form:

(a) a vertical compression by a factor of  $\frac{1}{2}$ , a reflection in the  $x$ -axis and a translation 3 units right.  $\rightarrow a$  is  $\ominus$

$$y = -\frac{1}{2} f(x - 3)$$

(b) a vertical stretch by a factor of 3, a horizontal stretch by a factor of 2, a translation left 5 and up 4, and a reflection in the  $y$ -axis.  $a = 3$   $k = -\frac{1}{2}$   $d = 5$   $c = 4$

$$y = 3 f(-\frac{1}{2}(x + 5)) + 4$$

**Practice Transformations Given an Equation**

Graph each of the following functions by:

a) Graphing the base function first. ( $y = x^2, y = \sqrt{x}, y = x^3, y = |x|, y = \frac{1}{x}$ )  $y = af(k(x-d)) + c$

b) Listing the transformations.

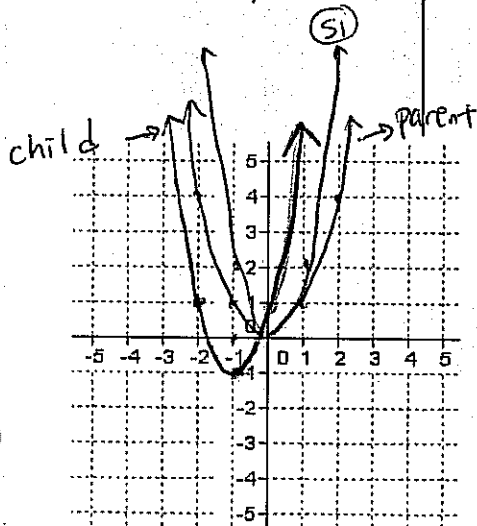
c) Applying the transformations to the base function.

$$y = x^2$$

1)  $y = 2(x+1)^2 - 1$

$$y = 2f(x+1)^2 - 1$$

- ① Vertical stretch by 2
- ② shift 1 to the left
- ③ shift down by 1



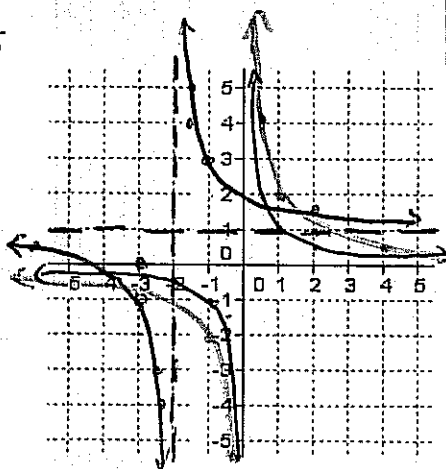
parent =  $\frac{1}{x}$

2)  $y = \frac{2}{x+2} + 1$

$$y = 2f(\frac{1}{x+2}) + 1$$

- ① Vertical stretch by 2
- ② shift 2 to the left
- ③ shift up 1 unit

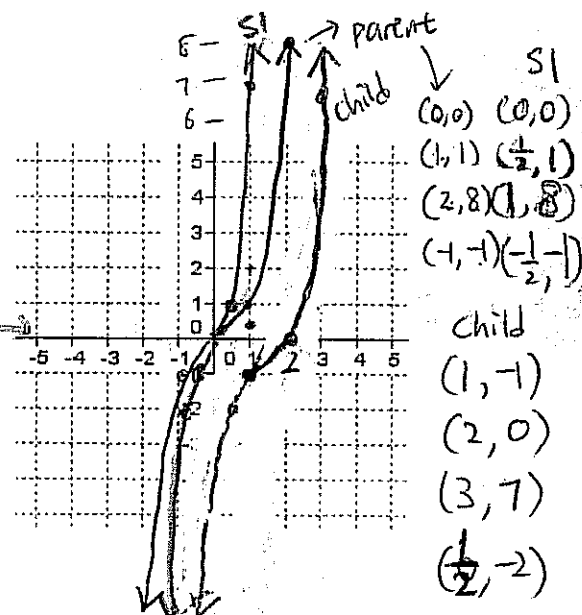
affects asymptotes



$$f(x) = x^3$$

3)  $y = (2x-2)^3 - 1$   $y = f(2(x-1))^3 - 1$

- ① Horizontal stretch by  $\frac{1}{2}$
- ② shift 1 to the right
- ③ shift 1 down



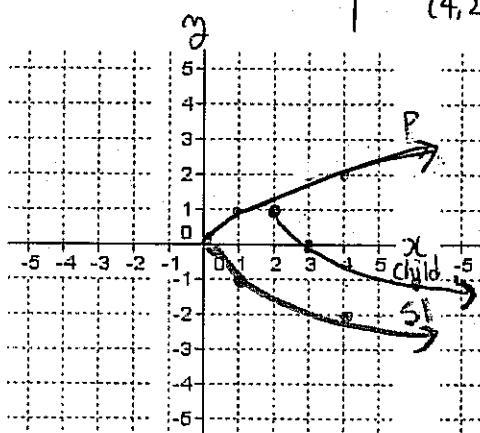
$$y = \sqrt{x}$$

$$4) y = -\sqrt{x-2} + 1$$

$$a = -1, k = 1, d = 2$$

$$c = 1$$

- ① reflect on x axis
- ② shift to the right by 2
- ③ shift up by 1



$$y = \sqrt{x}$$

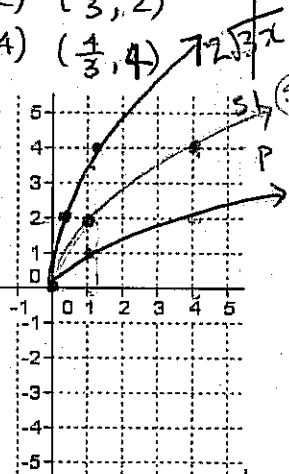
$$5) y = 2\sqrt{3x}$$

$$y = 2f(3(x))$$

$$a = 2, k = 3, d = 0, c = 0$$

- ① Vertical stretch by 2
- ② horizontal compression by  $\frac{1}{3}$

- ③ (1,1) (1,2) ( $\frac{1}{3}, 2$ )
- (4,2) (4,4) ( $\frac{1}{3}, 4$ )

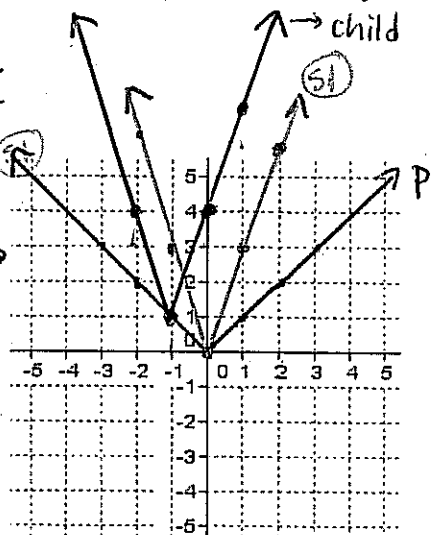


$$f(x) = |x|$$

$$6) y = 3|x+1| + 1$$

$$a = 3, k = -1, d = -1, c = 1$$

- ① Vertical stretch by 3
- ② shift to the left by 1
- ③ shift UP 1



$$f(x) = |x| \rightarrow \text{parent}$$

$$f(x) = x^3 \rightarrow \text{parent}$$

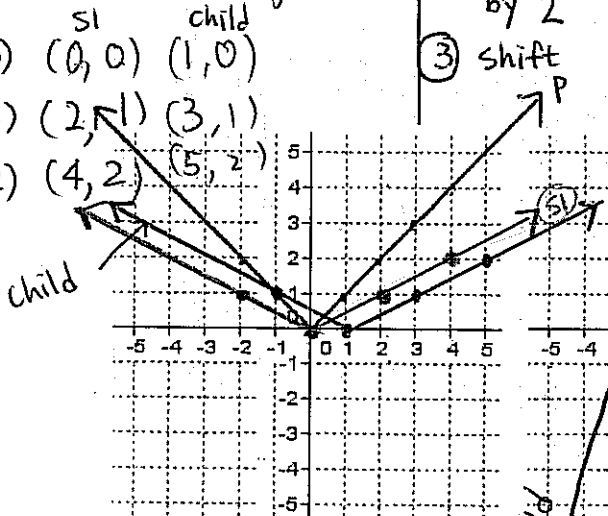
$$7) y = \left| \frac{1}{2}x - \frac{1}{2} \right|$$

$$y = \left| \frac{1}{2}(x-1) \right|$$

$$a = 1, k = \frac{1}{2}, d = 1, c = 0$$

- ① Horizontal stretch by 2
- ② shift 1 to the right

- P (0,0) (0,0) (1,0)
- SI (1,1) (2,1) (3,1)
- child (2,2) (4,2) (5,2)

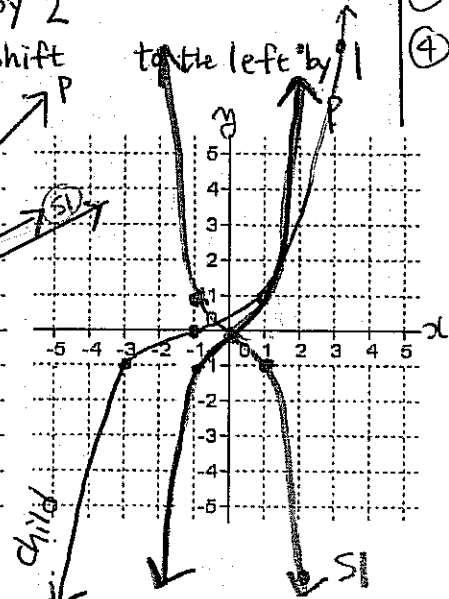


$$8) y = -\left(-\frac{1}{2}(x+1)\right)^3, c = 0$$

$$a = -1, k = -\frac{1}{2}, d = -1$$

- ① reflect it on x axis
- ② reflect it on y axis and horizontal stretch by 2
- ③ shift

to the left by 1



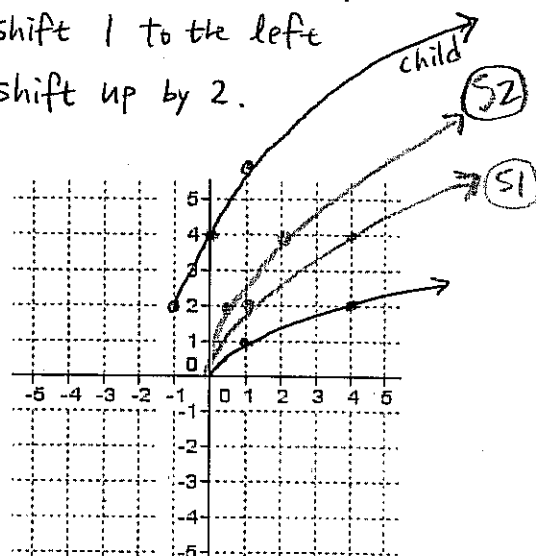
$$f(x) = \sqrt{x}$$

$$9) y = 2\sqrt{2x+2} + 2, y = 2\sqrt{2(x+1)} + 2$$

$$y = 2f(2(x+1)) + 2$$

$$a = 2, k = 2, d = -1, c = 2$$

- ① Vertical stretch by 2
- ② horizontal compression by  $\frac{1}{2}$
- ③ shift 1 to the left
- ④ shift up by 2



**Practice Transformations Given a Graph**

List the transformations.

Apply the transformations to key points on the graph.

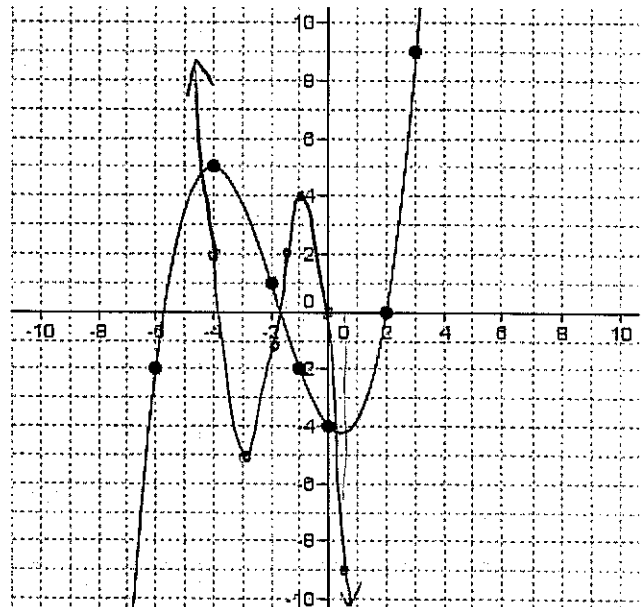
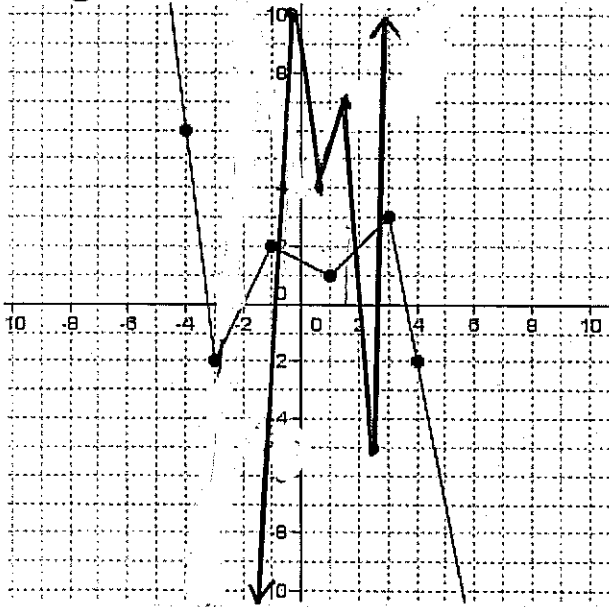
1)  $y = 3g(-2(x-1)) + 1$   $a=3, k=-2$   
 $d=1, c=1$

2)  $y = -f(2(x+1))$

① Vertically stretch by 3.

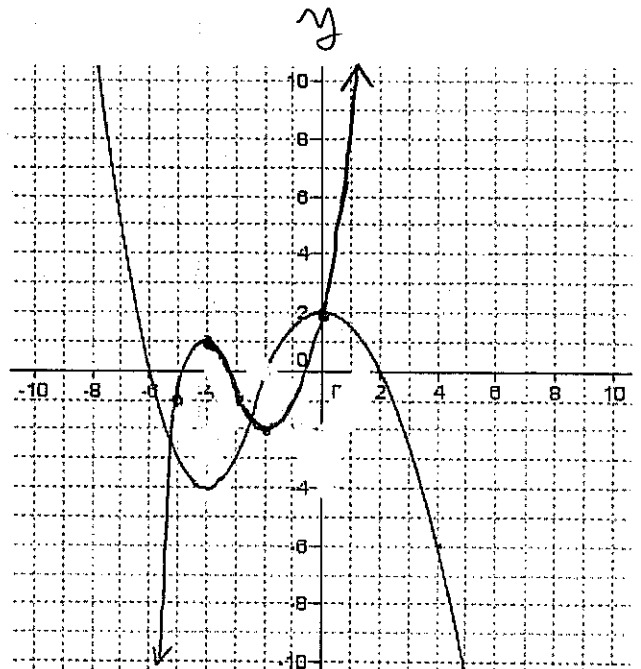
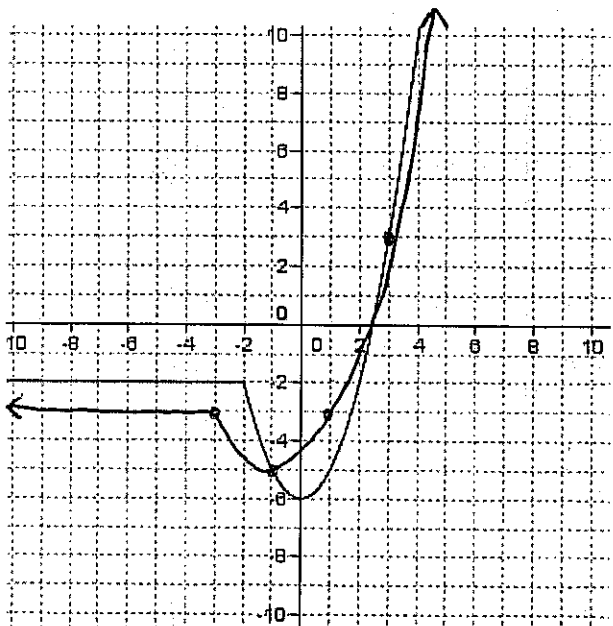
② reflect it on y axis and horizontally compressed by  $\frac{1}{2}$

③ shift 1 to the right and 1 up.



3)  $y = \frac{1}{2}h(x+1) - 2$

4)  $y = -\frac{1}{2}f(2x+4) - 1$



$$1) y = 3g(-2(x-1)) + 1$$

$$a = 3, \quad k = -2, \quad d = 1, \quad c = 1$$

① Vertically stretch by 3.

② Reflect it on y-axis and horizontally compress by  $\frac{1}{2}$

③ Shift 1 to the right and 1 up.

Parent	S1	S2	S2.5	S3
$(-4, 6)$	$(-4, 18)$	$(4, 18)$	$(2, 18)$	$(3, 19)$
$(-3, -2)$	$(-3, -6)$	$(3, -6)$	$(\frac{3}{2}, -6)$	$(2.5, -5)$
$(-1, 2)$	$(-1, 6)$	$(1, 6)$	$(\frac{1}{2}, 6)$	$(1.5, 7)$
$(1, 1)$	$(1, 3)$	$(-1, 3)$	$(-\frac{1}{2}, 3)$	$(0.5, 4)$
$(3, 3)$	$(3, 9)$	$(-3, 9)$	$(-\frac{3}{2}, 9)$	$(-0.5, 10)$
$(4, -2)$	$(4, -6)$	$(-4, -6)$	$(-2, -6)$	$(-1, -5)$

$$2) y = -f(2(x+1))$$

$$a = -1, \quad k = 2, \quad d = -1, \quad c = 0$$

Parent	S1	S2	S3
$(-6, -2)$	$(-6, 2)$	$(-3, 2)$	$(-4, 2)$
$(-4, 5)$	$(-4, -5)$	$(-2, -5)$	$(-3, -5)$
$(-2, 1)$	$(-2, -1)$	$(-1, -1)$	$(-2, -1)$
$(-1, -2)$	$(-1, 2)$	$(-\frac{1}{2}, 2)$	$(-1.5, 2)$
$(0, -4)$	$(0, 4)$	$(0, 4)$	$(-1, 4)$
$(2, 0)$	$(2, 0)$	$(1, 0)$	$(0, 0)$
$(3, 9)$	$(3, -9)$	$(\frac{3}{2}, -9)$	$(0.5, -9)$

① reflect it on x axis

② horizontal compression by  $\frac{1}{2}$

③ shift to the left by 1

$$3) y = \frac{1}{2} h(x+1) - 2 \quad a = \frac{1}{2}, d = -1, c = -2$$

① Vertically compress it by  $\frac{1}{2}$ .

② Shift to the left by 1 and down by 2.

Parent	(S1)	(S2)
$(-2, -2)$	$(-2, -1)$	$(-3, -3)$
$(0, -6)$	$(0, -3)$	$(-1, -5)$
$(2, -2)$	$(2, -1)$	$(1, -3)$
$(4, 10)$	$(4, 5)$	$(3, 3)$

$$4) y = -\frac{1}{2} f(2x+4) - 1$$

$$a = -\frac{1}{2}, k = 2, d = -2,$$

$$y = -\frac{1}{2} f(2(x+2)) - 1$$

$$c = -1$$

① reflect it on  $x$  axis and vertically compress by  $\frac{1}{2}$

② Horizontal compression by  $\frac{1}{2}$

③ Shift to the left by 2 and down by 1

Parent	(S1)	(S2)	(S3)
$-6, 0$	$-6, 0$	$-3, 0$	$-5, -1$
$-4, -4$	$-4, +2$	$-2, +2$	$-4, 1$
$-2, 0$	$-2, 0$	$-1, 0$	$-3, -1$
$0, 2$	$0, -1$	$0, -1$	$-2, -2$
$4, -6$	$+4, +3$	$+2, +3$	$0, 2$