March 23 Park MCR3U Unit 3: Quadratic Function

Youtube: "Maximum and Minimum Values of Quadratic function" by Patrick

*submit assignment to parkr@hdsb.ca Max/Min Values | MCR3U

* Quadratic Function? Any function, which contains x^2 term.
either shape $\cap$ or $\cup$

* Vertex of a quadratic function can be found by
① Completing the square method (similar to factoring)
② Graphing (or table of values) by using Unit 2: $y = af(x-d) - c$
③ Partial Factoring

* 3 forms of quadratic equations
① Standard form $f(x) = ax^2 + bx + c \rightarrow c = y$ intercept
② Vertex form $y = a(x-h)^2 + k \rightarrow$ vertex $= (h, k)$
* If a is positive, then shape is $\cup$ → k is minimum value.
* If a is negative, then shape is $\cap$ → k is maximum value.
③ Factored Form $f(x) = a(x-r)(x-s) \rightarrow x$ intercepts are r and s.

* Completing the square is a process that you change a quadratic equation from standard form into vertex form.

e.g) 2a) Step 0.5: Factor x^2 term so that it only has 1 as coefficient

Step 1: Divide x term's coefficient by 2 then square that number
Step 2: Add that number and minus that number at the same time.
Step 3: change the equation into perfect square form.
Step 4: Add or subtract the remaining numbers.

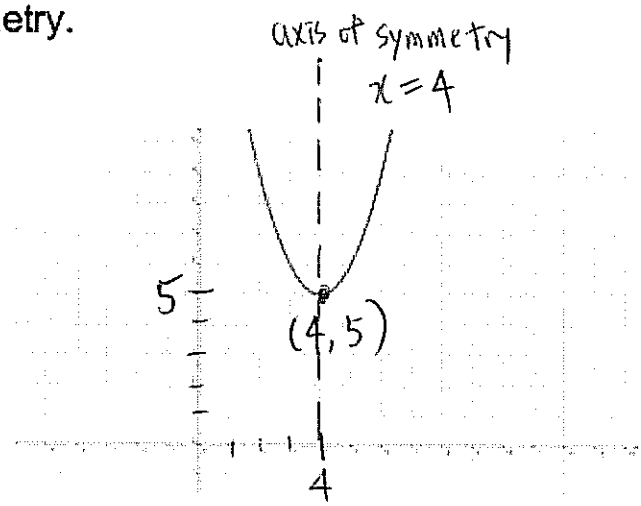
March 23

Unit 2 - Quadratics

Review of Max/Min Values

1) For each, state the vertex, the max/min value and the axis of symmetry.

a)



Vertex = (4, 5)

Min value = 5

Axis of symmetry ⇒ $x = 4$

↓ = middle line

→ $y = af(k(x-d)) + c$

b) $f(x) = 5x^2 + 2$

Since you can not factor nor completing the square, you will just use Unit 2 skill.

$a = 5$ and $c = 2$ so vertex = (0, 2)

Since $a = 5$ → Min = 2 → ∪

Axis of symmetry ⇒ $x = 0$

c) $f(x) = -4(x - 7)^2 - 3$

$f(x) = a(x-h)^2 + k$ (h, k)

Vertex = (7, −3)

∩ Since a = negative → Max value is −3.

Axis of symmetry ⇒ $x = 7$

$\frac{8x}{2x} = a$ → $a = 4$

2) Find the max/min value of each by completing the square.

a) $f(x) = x^2 + 8x + 1$

$(x+a)^2 = x^2 + 2ax + a^2$

$f(x) = x^2 + 8x + 16 - 16 + 1$

$a = 4$ → $x^2 + 2ax + a^2$

$f(x) = (x+4)^2 - 15$ → ∪

Vertex = (−4, −15)

∴ Min value is −15.

b) $f(x) = -x^2 - 12x + 5$

$f(x) = -(x^2 + 12x - 5)$ $\quad (12 \div 2)^2$

$f(x) = -(x^2 + 12x + 36 - 36 - 5)$

$= -\{(x+6)^2 - 41\}$

$= -(x+6)^2 + 41$

Vertex $= (-6, 41)$ $\quad$ Max $= 41$

c) $f(x) = 2x^2 + 12x - 3$ $\quad (6 \div 2)^2$

$f(x) = 2(x^2 + 6x) - 3$

$f(x) = 2(x^2 + 6x + 9 - 9) - 3$

$= 2[(x+3)^2 - 9] - 3$

$= 2(x+3)^2 - 18 - 3$

$= 2(x+3)^2 - 21$

Vertex $(-3, -21)$ $\quad$ Min $= -21$

d) $f(x) = -x^2 + 3x + 7$ $\quad \left(-\frac{3}{2}\right)^2 = \frac{9}{4}$

$f(x) = -(x^2 - 3x) + 7$

$= -(x^2 - 3x + \frac{9}{4} - \frac{9}{4}) + 7$

$= -(x - \frac{3}{2})^2 + \frac{9}{4} + 7$ $\quad \frac{7}{1} = \frac{4 \times 7}{4} = \frac{28}{4}$

$= -(x - \frac{3}{2})^2 + \frac{9+28}{4}$

$= -(x - \frac{3}{2})^2 + \frac{37}{4}$ $\rightarrow$ Vertex $= (\frac{3}{2}, \frac{37}{4})$ $\quad$ Max $= \frac{37}{4}$

e) $f(x) = \frac{1}{2}x^2 + 6x + 1$ $\quad (12 \div 2)^2$

$f(x) = \frac{1}{2}(x^2 + 12x + 2)$

$f(x) = \frac{1}{2}(x^2 + 12x + 36 - 36 + 2)$

$f(x) = \frac{1}{2}[(x+6)^2 - 34]$

$f(x) = \frac{1}{2}(x+6)^2 - 17$ $\quad$ Min $= -17$

Vertex $= (-6, -17)$

f) $f(x) = -\frac{2}{3}x^2 + 8x + 5$

$8 \div -\frac{2}{3} = 8^4 \times -\frac{3}{2} = -12$

$f(x) = -\frac{2}{3}(x^2 - 12) + 5$

$f(x) = -\frac{2}{3}(x^2 - 12 + 36 - 36) + 5$

$f(x) = -\frac{2}{3}[(x-6)^2 - 36] + 5$ $\quad a^2 - 2ab + b^2 = (a-b)^2$

$= -\frac{2}{3}[x-6)^2 + 24 + 5$

$= -\frac{2}{3}(x-6)^2 + 29$

so vertex $= (6, 29)$

$\therefore$ Max $= 29$ $\rightarrow$

* In grade 10 Math, you learned $x^2 + y^2 = r^2$ (Equation of a circle)

Fact: The equation $(x - a)^2 + (y - b)^2 = r^2$ is the equation of a circle with radius r and centre (a,b).

Find the radii and centres of the following circles.

a) $x^2 - 8x + y^2 + 6y + 16 = 0$

b) $x^2 + 14x + y^2 - 4y = -37$

Hwk. pg. 31 #1-3

* Unit 3 test will be on April 2 (Thursday)