MPM2D March 9 **Midpoint and Median**

Recall: Find the equation of a line that passes through $(1,1)$ and $(5,9)$.

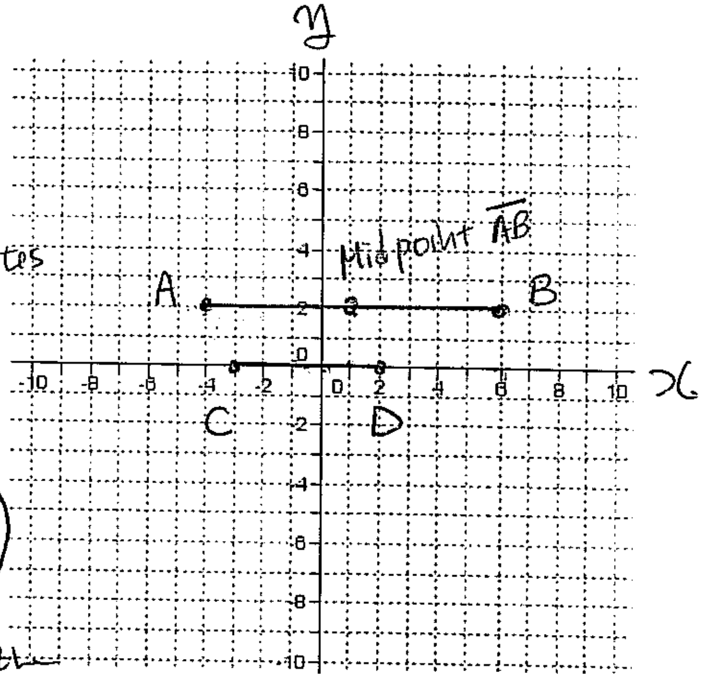
Definition:

Midpoint – A point that divides a line segment into two equal line segments.

Investigation:

How can you determine the coordinates of a midpoint?

1. Plot and label the line segment defined by each pair of endpoints.
   a. $A(-4,2)$ and $B(6,2)$
   b. $C(-3,0)$ and $D(2,0)$



Question: What property do the line segments have in common?

* Both lines contained same y coordinates so you just had to add x_1 and x_2 then divide it by 2.

* Midpoint of $\overline{AB}$ = $(1, 2)$ because

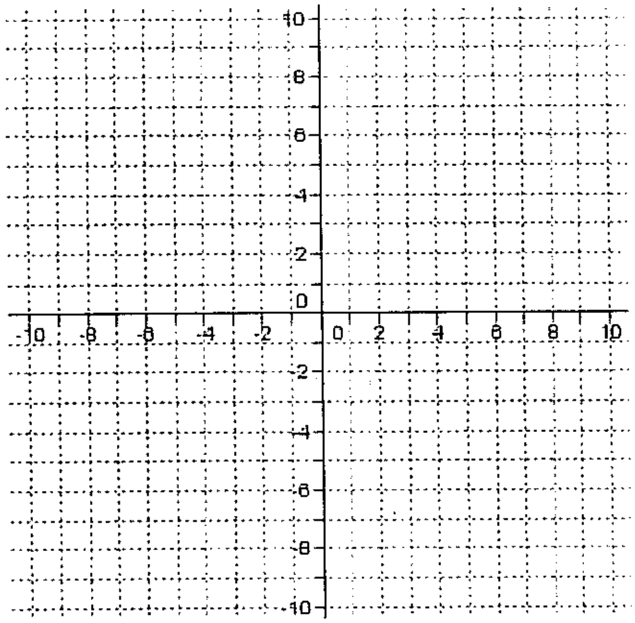
* Midpoint of $\overline{CD}$ = $(-\frac{1}{2}, 0)$

y coordinate did not change between the two points. $\frac{x_2+x_1}{2} = \frac{6-4}{2} = \frac{2}{2} = 1$

2. Count squares to determine the coordinates of the midpoint of each segment. Label each midpoint with its coordinates.

Question: How are the coordinates of the midpoint of each line segment related to the coordinates of its endpoints?

3. Plot and label the line segment defined by each pair of endpoints.
    a. G(−4, 2) and H(−4, 6)
    b. J(−1, 4) and K(−1, −2)

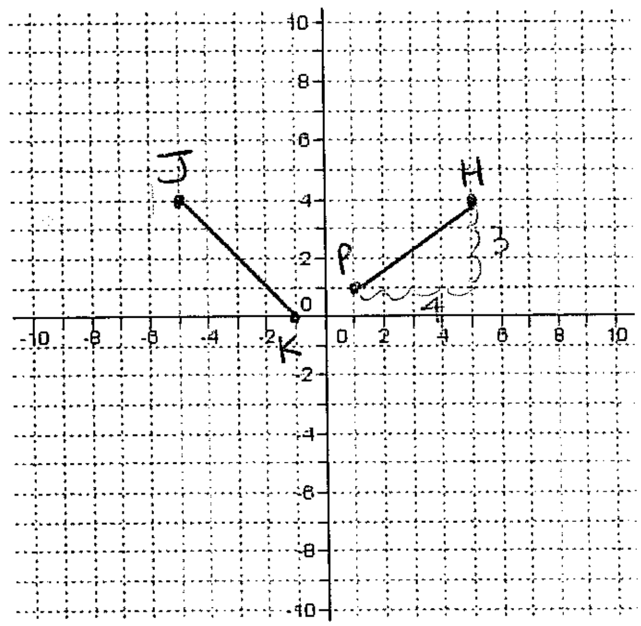


Question: What property do the line segments have in common?

4. Determine the coordinates of the midpoint of each line segment in step 3. Label each midpoint with its coordinates.

Question: How are the coordinates of the midpoint of each line segment related to the coordinates of its endpoints?

5. Plot and label the line segment defined by each pair of endpoints.
    a. P(1, 1) and H(5, 4)
    b. J(−5, 4) and K(−1, 0)



Determine the coordinates of the midpoint of each line segment in step 5. Describe how you calculated these coordinates.

$$x \text{ coordinate of } \overline{PH} = \frac{\overset{x_1}{5}+\overset{x_2}{1}}{2} = \frac{6}{2} = 3$$

$$y \text{ coordinate } \overline{PH} = \frac{\overset{y_1}{4}+\overset{y_2}{1}}{2} = 2.5$$

$$\text{Midpoint } \overline{PH} = (3, 2.5)$$

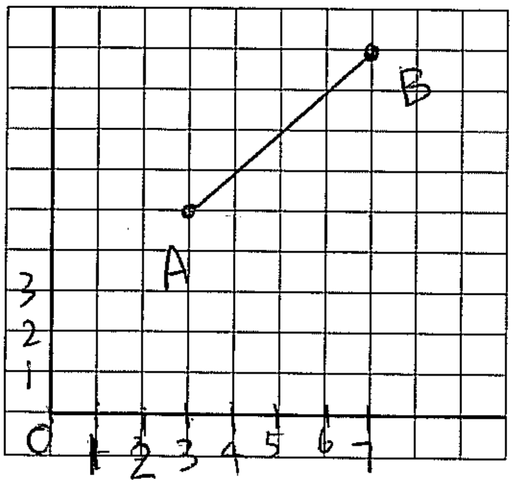
6. Can you write the formula for the coordinates of the midpoint of a line segment related to the coordinates of the endpoints?

The formula for a midpoint with endpoints (x_1, y_1) and (x_2, y_2) is:

$$\boxed{M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)}$$

Example 1: Find a midpoint.

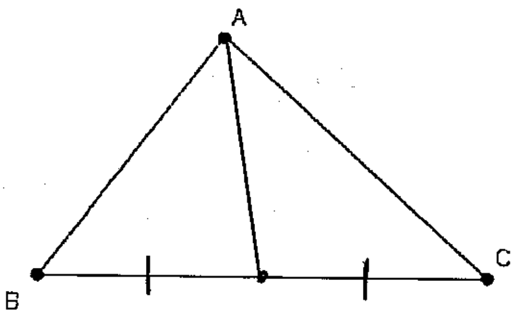
A city has two hospitals, at coordinates $A(\overset{x_1}{3}, \overset{y_1}{5})$ and $B(\overset{x_2}{7}, \overset{y_2}{9})$. The city wants to build a new ambulance station halfway between the two hospitals. Determine the coordinates of this location.



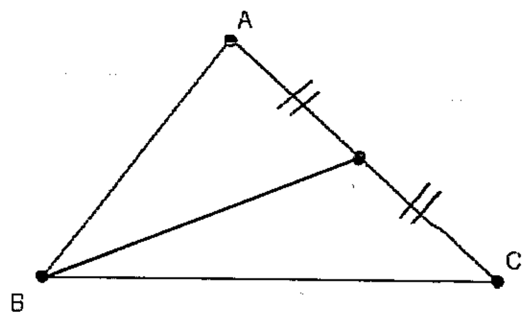
$$M = \left(\frac{3+7}{2}, \frac{5+9}{2}\right)$$

$$M = (5, 7)$$

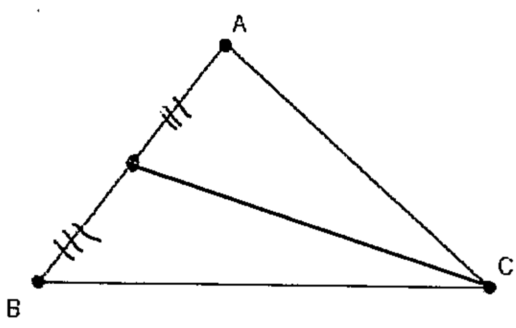
Median – A line segment joining a vertex of a triangle to the midpoint of the opposite side.



Draw the median from A



Draw the median from B



Draw a median from C

Example 2: Median of a Triangle

$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad x_2 \; y_2$

Determine an equation for the median from vertex C for the triangle with vertices C(5, 2), A(−3, 3), and B(2, −5).

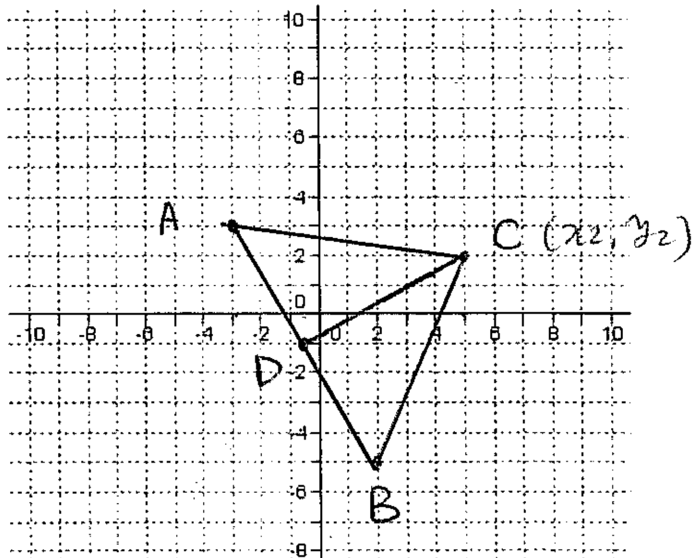
$x_1 \; y_1 \quad x_2 \; y_2$

$$\text{Midpoint } \overline{AB} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$= \left(\frac{-3+2}{2}, \frac{3-5}{2}\right)$$

$$= \left(-\frac{1}{2}, \frac{-2}{2}\right)$$

$$\text{Mid } \overline{AB} = \left(\underset{x_1}{-\frac{1}{2}}, \underset{y_1}{-1}\right)$$



$$\underset{\text{of } \overline{CD}}{m} = \frac{y_2-y_1}{x_2-x_1} = \frac{(2 - -1)}{(5 - -\frac{1}{2})} = \frac{3}{\frac{11}{2}} = 3 \div \frac{11}{2} = 3 \times \frac{2}{11} = \frac{6}{11}$$

$$= \frac{3}{5.5} = 0.545 \text{ or } \frac{6}{11} \quad \therefore y = \frac{6}{11}x + b \quad \text{Sub C point into}$$

Right Bisector - the line that passes through the midpoint of a line segment and intersects it at a 90° angle.

$$\text{the } y = \frac{6}{11}x + b \longrightarrow 2 = \frac{6}{11}(5) + b$$

$$2 = \frac{30}{11} + b \longrightarrow 2 - \frac{30}{11} = b \rightarrow b = \frac{22-30}{11} = -\frac{8}{11}$$

$\therefore$ The equation of a median $(= \overline{CD})$ is $y = \frac{6}{11}x - \frac{8}{11}$

MPM2D
March 9 HW

Midpoint and Median Worksheet

1. Determine the midpoint of each line segment from the given endpoints.
   a) $(4,4)$ and $(10,-6)$
   b) $(-5,3)$ and $(1,-1)$
   c) $(2,7)$ and $(2,-2)$
   d) $(4.2,1.9)$ and $(3.4,-4.4)$
   e) $\left(\frac{1}{2},\frac{5}{2}\right)$ and $\left(\frac{3}{2},-\frac{9}{2}\right)$

2. For a line segment PQ, one endpoint is $Q(8,3)$ and the midpoint is $M(3,6)$. Determine the coordinates of the other endpoint, P.

3. Determine the equation of the perpendicular bisector for the line segment with endpoints $A(-2,9)$ and $B(8,3)$

4. The centre of a circle has coordinates $(0,0)$. The endpoint of a diameter of the circle has coordinates $(3,-4)$. What are the coordinates of the other endpoint of the diameter?

5. Triangle ΔDEF has vertices $D(-2,0)$, $E(4,-3)$ and $F(8,8)$
   a) Draw ΔDEF.
   b) Draw and determine the equation of the median from D to the midpoint of EF.

6. The vertices of ΔABC are $A(4,4)$, $B(-6,2)$, and $C(2,0)$. Find an equation in slope y-intercept form for the median from vertex A.

7. For the triangle with vertices $P(-2,0)$, $Q(4,6)$ and $R(5,-3)$, find an equation for the median from
   a) Vertex P
   b) Vertex Q

Thinking

8. Write an expression for the coordinates of the midpoint of the line segment with endpoints $P(a,b)$and $Q(3a,2b)$. Explain your reasoning.

9. Decide whether each statement is always true, sometimes true, or never true. Justify your answers (diagrams may be useful).
   a) Two line segments with the same midpoint have the same length.
   b) Two parallel line segments have the same midpoint.
   c) The midpoint of a line segment is the only point that divides it into two equal parts.
   d) A point equidistant from the endpoints of a line segment is the midpoint of the line segment.

10. In three dimensions, the location of a point can be represented by the ordered triple (x, y, z).
   a) Find the coordinates of the midpoint of the line segment with endpoints $A(2,3,1)$ and $B(6,7,5)$.
   b) Write an expression for the coordinates of the midpoint of the line segment with endpoints (x_1, y_1, z_1) and (x_2, y_2, z_2).

11. A line segment has endpoints $A(2,1)$ and $B(11,19)$.
   a) Find the coordinates of the two points that divide the line segment into three parts. Check your answer with a graph.
   b) Describe the method that you used in part a).

12. The endpoints of line segment PQ are $P(3,-4)$ and $Q(11,c)$. The midpoint of PQ is $M(d,3)$. Find the values of c and d.

Challenge:

13. Determine the equation for the right bisector of the line segment with endpoints $P(-5,-2)$ and $Q(3,6)$.

Answers:

1. a) $(7,-1)$ b) $(-2,1)$ c) $\left(2,\frac{5}{2}\right)$ d) $(3.8,-1.25)$ e) $(1,-1)$
2. $P(-2,9)$
3. $y=\frac{5}{3}x+1$
4. $(-3,4)$
5. $y=\frac{5}{16}x+\frac{5}{8}$
6. $y=\frac{1}{2}x+2$
7. a) $y=\frac{3}{13}x+\frac{6}{13}$ b) $y=3x-6$
8. $(2a, 1.5b)$, these coordinates are the mean of the x-coordinates of the endpoints and the mean of the y-coordinates of the endpoints.
9.
   a) Sometimes true: Line segments can bisect each other without being equal in length.
   b) Never true: Parallel lines have no points in common.
   c) Always true: The midpoint is the only point that is both on the line segment and equidistant from the endpoints.
   d) Sometimes true: The midpoint of a line segment is equidistant from the endpoints but so is every other point on the right bisector of the line segment (next lesson)
10. a) $(4,5,3)$ b) $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right)$
11. $(5,7)$ and $(8,13)$
12. $c=10$, $d=7$
13. $y=-x+1$