* In grade 10 Math, you learned $x^2 + y^2 = r^2$ (Equation of a circle)

March 25

Fact: The equation $(x - a)^2 + (y - b)^2 = r^2$ is the equation of a circle with radius r and centre (a,b).

Find the radii and centres of the following circles.

a) $x^2 - 8x + y^2 + 6y + 16 = 0 \longrightarrow (x-a)^2 + (y-b)^2 = r^2$

$x^2 - 8x + y^2 + 6y = -16$

$x^2 - 8x + (-8\div2)^2 - 16 + y^2 + 6y + (6\div2)^2 - 9 = -16$

$(x-4)^2 + (y+3)^2 = -16 + 16 + 9$ $\therefore$ centre $(4, -3)$

$(x-4)^2 + (y+3)^2 = 9 \Rightarrow \sqrt{r^2} = \sqrt{9} \Rightarrow r = \pm 3$ $\therefore$ radius $= 3$

b) $x^2 + 14x + y^2 - 4y = -37$

$x^2 + 14x + (14\div2)^2 - 49 + y^2 - 4y + (-4\div2)^2 - 4 = -37$

Hwk. pg. 31 #1-3

$(x+7)^2 + (y-2)^2 = -37 + 49 + 4$

$(x+7)^2 + (y-2)^2 = 16 \Rightarrow \sqrt{r^2} = \sqrt{16} \Rightarrow r = \pm 4$

$\therefore$ radius $= 4$ and centre $= (-7, 2)$

* Unit 3 test will be on April 2 (Thursday)

March 25 Test will be Wed April 1st

Max/Min Applications

1) The height of a ball, in metres, is given by $h(t) = -5t^2 + 20t$ where t is the number of seconds. What is the maximum height the ball reaches and when does it get there? When would the ball land?

"Max height" = Max Y value? or vertex? → $t=?$ when $h=0$

$h(t) = -5t^2 + 20t$ → Change it to vertex form

$= -5(t^2 - 4t)$

$= -5(t^2 - 4t + (-4 \div 2)^2 - 4)$

$= -5(t-2)^2 + 20$

∴ vertex = $(2, 20)$ (t, h)

∴ Max height is 20 m when time is 2 seconds. (= After 2 seconds, the ball reaches 20m height)

* when $h=0$, $t=?$

$0 = -5t^2 + 20t$

$0 = -5t(t-4)$

$\therefore t = 4, 0$

∴ The ball would land after 4 seconds.

2) The student council sold 400 tickets at $11 each for their talent show. Through a survey they found that for every $1 increase in price, 20 fewer people would attend. What ticket price will maximize their revenue?

Revenue = Y price = x because price (input) causes revenue (output)

Let x represent # of $ increase

Revenue = Price × quantity sold

$F(x) = (11 + 1x)(400 - 20x)$

$= 4400 - 220x + 400x - 20x^2$

$= -20x^2 + 180x + 4400$

$= -20(x^2 - 9x) + 4400$

$= -20\left[x^2 - 9x + \left(\frac{-9}{2}\right)^2 - \left(\frac{81}{4}\right)\right] + 4400$

$= -20\left(x - \frac{9}{2}\right)^2 + 405 + 4400$

$= -20\left(x - \frac{9}{2}\right)^2 + 4805$

∴ vertex = $\left(\frac{9}{2}, 4805\right)$

Price increase ↑ ↓ Revenue

∴ At $15.50, the revenue will be maximized.

3) Bob makes scarves for a craft show that cost $6 each to make. He plans to sell them for $10 each. Last year he sold 40 scarves. After doing some research he found that if he increases the price by $0.50 he would lose 4 customers. What price should he charge to maximize profit and what would the profit be?

Let x represent the number of 50 cents price change

Unit Profit = Unit price − Unit cost

= 10 − 6 = 4 so $4 per scarf

Profit = profit per scarf × quantity sold

$P(x) = (4 + 0.5x)(40 - 4x)$

* Use partial factoring → set $y = 0$

$0 = (4 + 0.5x)(40 - 4x)$

$y = 0$ when $(4 + 0.5x) = 0$ or $(40 - 4x) = 0$

$4 + 0.5x = 0$

$0.5x = -4$

$x = -8$ or

$40 - 4x = 0$

$-4x = -40$

$x = 10$

$\frac{10 + (-8)}{2} = 1$

x coordinate of vertex

Sub $x = 1$ → $P(x) = (4 + 0.5)(36)$

$P(x) = 162$

∴ At $10.50 the profit will be maximized to $162.

4) Find two numbers that have a sum of 14 and whose product is a maximum.

(A) $x + y = 14$

(B) $xy = \max$

Hwk. Pg. 31 # 5 - 7, 11