March 27

Max/Min Applications | MCR3U

3) Bob makes scarves for a craft show that cost $6 each to make. He plans to sell them for $10 each. Last year he sold 40 scarves. After doing some research he found that if he increases the price by $0.50 he would lose 4 customers. What price should he charge to maximize profit and what would the profit be?

Let x represent the number of 50 cents price change

Unit Profit = Unit price − Unit cost

= 10 − 6 = 4 so $4 per scarf

Profit = profit per scarf × quantity sold

$P(x) = (4+0.5x)(40-4x)$

* Use partial factoring → set $y=0$

$0 = (4+0.5x)(40-4x)$

$y=0$ when $(4+0.5x)=0$ or $(40-4x)=0$

$4+0.5x=0$
$0.5x=-4$
$x=-8$ or

$40-4x=0$
$-4x=-40$
$x=10$

$\frac{10+(-8)}{2} = 1$

x coordinate of vertex

Sub $x=1$ → $P(x) = (4+0.5)(36$

$P(x) = 162$

∴ At $10.50 the profit will be maximized to $162.

4) Find two numbers that have a sum of 14 and whose product is a maximum.

* Let x represent the first number
* Let y " " second "

(A) $x+y=14$ → $y=14-x$

(B) $xy = \max$

(B) $x(14-x)=0$

x intercepts are 0 or 14

x coordinate of vertex $= \frac{0+14}{2} = 7$

Sub $x=7$ into (A)

$7+y=14$

$y=14-7=7$

∴ The two numbers are 7 and 7.

Hwk. Pg. 31 # 5 - 7, 11

March 27 Friday Unit Test will be Wed (April 8th)

Applications of Quadratic Equations

1) Two numbers have a difference of 4 and the sum of their squares is 208. What are the numbers?

Let x represent first number.
" y " second "

$x - y = 4$ — Ⓐ

$x^2 + y^2 = 208$ — Ⓑ

Rearrange Ⓐ $-y = 4 - x$

$\therefore y = x - 4$ ← $\times(-1)$ both sides

Sub $y = x-4$ into Ⓑ

Ⓑ. $x^2 + (x-4)^2 = 208$

$x^2 + x^2 + 16 - 8x = 208$

$2x^2 - 8x - 192 = 0$

* Divide by 2 both sides

$x^2 - 4x - 96 = 0$

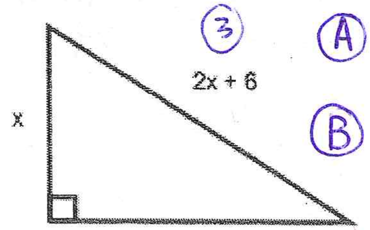
$ac = -96, \; b = -4$

$-12 \times 8 = -96$

$-12 + 8 = -4$

$(x-12)(x+8) = 0$ $\therefore x = 12, -$

2) The perimeter of a right triangle is 60 cm. The lengths of 2 sides are shown. Find the length of all 3 sides. Let x represent side #1 and y represent sid



③ ① y ②

Ⓐ — $x + 2x + 6 + y = 60$ (perimeter)

Ⓑ — $x^2 + y^2 = (2x+6)^2$ (Pythagorian theorem

Ⓐ' $3x + y = 54 \rightarrow y = 54 - 3x$ → sub into

Ⓑ $x^2 + (54-3x)^2 = (2x+6)^2$

$x^2 + 2916 + 9x^2 - 324x = 4x^2 + 36 + 24x$

$6x^2 - 348x + 2880 = 0$

* Divide by 6 both sides

$x^2 - 58x + 480 = 0$

$ac = 480, \; b = -$

$-10 \times -48 = 480$

$-10 + -48 = -58$

$(x-10)(x-48) = 0$

$\therefore x = 10, 48$ → sub into Ⓐ'

$\therefore y = 54 - 3(10) = 24$ $\therefore y = 54 - 3$

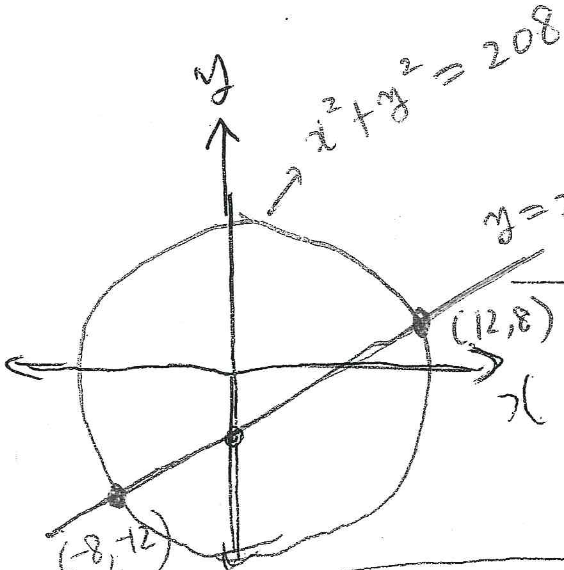
continue from March 27 #1

1) when $x=12 \longrightarrow 12-y=4$ Ⓐ

$$12-4=y$$
$$8=y \qquad \therefore (12,8)$$

when $x=-8 \longrightarrow -8-y=4$ Ⓐ

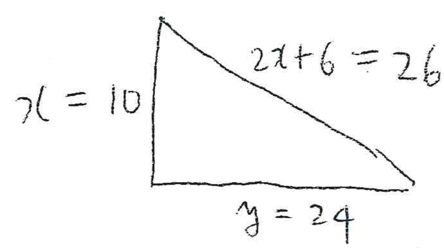
$$-y=4+8$$
$$-y=12$$
$$y=-12 \qquad \therefore (-8,-12)$$



$\longrightarrow$ That's why we have two solutions.

$\therefore$ The two numbers can be 12 and 8
or
-8 and -12

2) When $x=10 \longrightarrow y=24$



Side #1 : 10

Side #2 : 24

Side #3 : $2x+6 = 2(10)+6 = 26$

* We reject $(48,-90)$ because length of Δ can not become negative number.