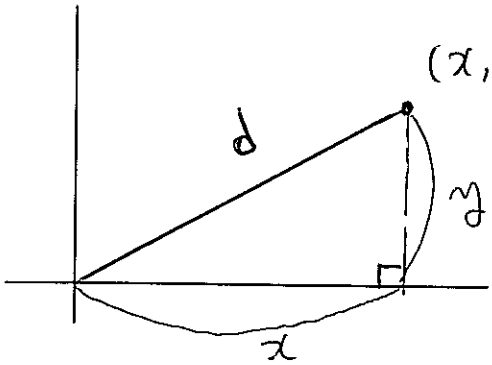
MPM2D March 11 **Distance from the Origin** Quiz on Friday



$d^2 = x^2 + y^2$ (pythagorian theorem)

$\sqrt{d^2} = \sqrt{x^2+y^2}$

$d = \sqrt{x^2+y^2}$

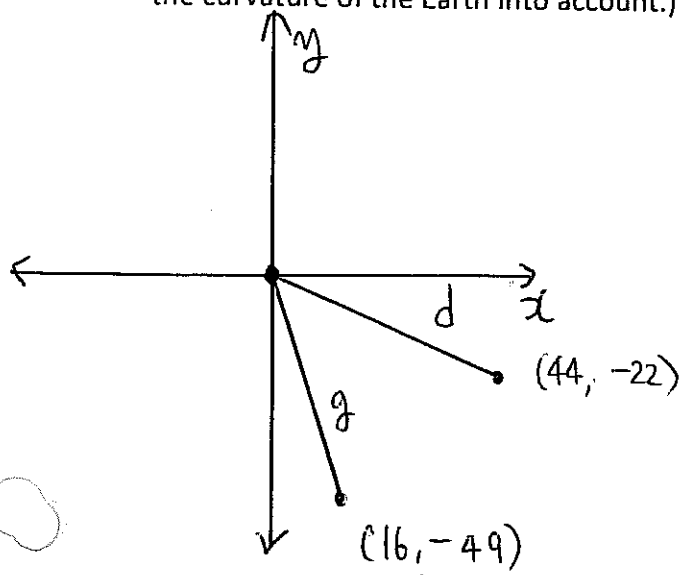
The distance, d, from the origin $(0, 0)$ to any point (x, y) is

$$\boxed{d = \sqrt{x^2 + y^2}}$$

Example 1 Find the distance from $P(3, -5)$ to the origin.

$$d = \sqrt{3^2 + (-5)^2} = \sqrt{9 + 25} = \sqrt{34}$$

Example 2 A boat's engine fails at sea. Coast guard ships are at $(44, -22)$ and at $(16, -49)$, where the first coordinate is the distance (in nautical miles) due east from the boat, and the second coordinate is the distance due south from the boat. If the boat is at the origin, which Coast Guard ship is closer to it? (A nautical mile (nm) is a unit of distance used in navigation. One nautical mile equals 1852 m. It is longer than a standard mile because it takes the curvature of the Earth into account.)



$d = \sqrt{44^2 + (-22)^2}$ (miles, miles)

$= \sqrt{1936 + 484}$

$= \sqrt{2420} = 49.2$ miles

$g = \sqrt{16^2 + (-49)^2}$

$= \sqrt{256 + 2401} = \sqrt{2657}$

$= 51.5$ miles

∴ The first Coast Guard ship is closer to the sinking ship.

Homework Questions: March 11

1. Plot each point and connect it to the origin with a straight line. Calculate the distance from the point to the origin.
   a. $(5, 12)$
   b. $(1, -1)$
   c. $(-7, -25)$

2. Which of these points is closest to the origin: X$(4, 6)$, Y$(-5, 5)$, or Z$(3, -7)$?

3. An airport control tower at $(0, 0)$ locates two airplanes on its radar screen. Plane A is at $(85, 95)$ and airplane B is at $(-64, 115)$. If both airplanes are approaching the airport at the same speed, which plane should be instructed to land first?

4. In an animated cartoon, a journalist morphs into a superhero. The computer animator describes the shape change in terms of pixel coordinate transformations. As a result of the transformation, the tip of the journalist's chin moves from a point with coordinates P$(12, 33)$ to R$(9, 45)$. How much farther is his chin from the origin after morphing?

5. In a video game, three animated characters are programmed to run out of a building and head in three different directions to escape a fire at the origin. After 2s, Animal is at A$(22, 18)$, Beast is at B$(-3, 35)$, and Creature is at C$(7, -29)$. Who ran the farthest?

Extension

6. A kite has vertices at A$(3, 3)$, B$(-2, 2)$, C$(-5, -5)$, and D$(2, -2)$
   a. Draw the kite on a grid
   b. Draw the kite's diagonals AC and BD
   c. Find the distance between points A and C
   d. Find the distance between points B and D

7. The Ministry of Natural Resources tagged and equipped two moose with tracking collars. After their release the moose head in different directions. Two hours later, one moose's location is given by $(6, 8)$. The other moose's position is given by $(-3, 5)$. Assuming that they were captured at $(0, 0)$, determine the difference between the distance they traveled.

8. Two of the vertices of ΔMNO are O$(0, 0)$ and M$(3, -4)$. What are possible coordinates for N if ΔMNO is
   a. Isosceles?
   b. Equilateral?

   Explain how you found your answers.

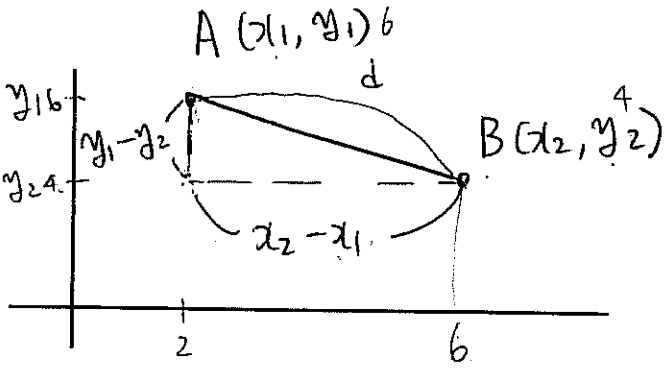
Answers:

1.a) 13 b) 1.4 c) 26 **2.** Y **3.** A **4.** 10.78 **5.** Beast
6. c) about 11.3 d) about 5.7 **7.** about 4.2

MPM2D

The Distance Between Any Two Points

March 11



$$d = \sqrt{(y_1 - y_2)^2 + (x_2 - x_1)^2}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

The distance between points $A(x_1, y_1)$ and $B(x_2, y_2)$ in the coordinate plane is

$$\boxed{d = \sqrt{(y_1 - y_2)^2 + (x_2 - x_1)^2}}$$

both are OK.

A **line segment** is the part of a line between two specific points, including the points themselves. The slope of a line segment is the same as the slope of the line containing the line segment.

Parallel line segments have the same slope.

The slope of two perpendicular line segments are negative reciprocal.

Example 1 Find the length of the line segments with these end points.

a) A(−1, 0) and B(5, 2)

$x_1\ y_1 \quad x_2\ y_2$

$$d = \sqrt{(0 - 2)^2 + (5 - -1)^2}$$

$$d = \sqrt{4 + 36} = \sqrt{40} = 6.3$$

b) G(−7, 8) and H(−7, −5)

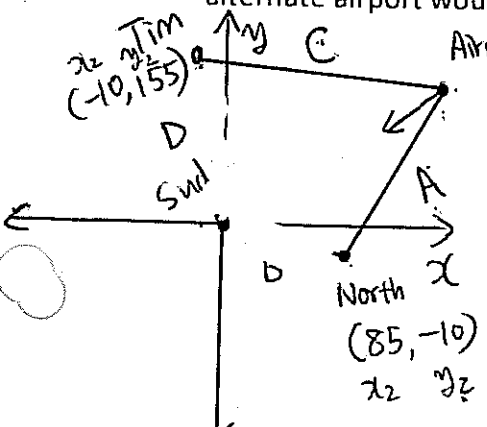
$x_1\ y_1 \quad x_2\ y_2$

$$d = \sqrt{(8 - -5)^2 + (-7 - -7)^2}$$

$$d = \sqrt{13^2 + 0^2} = \sqrt{13^2}$$

$$\therefore d = 13$$

Example 2 An airplane at coordinates (150, 136), which is heading for Sudbury (0, 0), has to be diverted from poor weather conditions to either North Bay (85, −10) or Timmins (−10, 155). If the airplane is carrying enough fuel to get to Sudbury, which alternate airport would it be safe to head for?



Airplane (150, 136)

$x_1\ y_1$

C

$$C = \sqrt{(-10 - 150)^2 + (155 - 136)^2}$$

$$C = \sqrt{25600 + 361} = \sqrt{25961}$$

$$\therefore C = 161.1$$

$$A = \sqrt{(85 - 150)^2 + (-10 - 136)^2}$$

$$A = \sqrt{4225 + 21316} = 160 \quad \therefore A = 160$$

$\therefore$ North Bay Airport is shorter distance than Timmins.

Example 3 A triangle has vertices at $A(-1,-1)$, $B(2,0)$, and $C(1,3)$. Find the lengths and slopes of the sides of the triangle. What kind of triangle is it?

Definitions:

Scalene

Isosceles

Equilateral

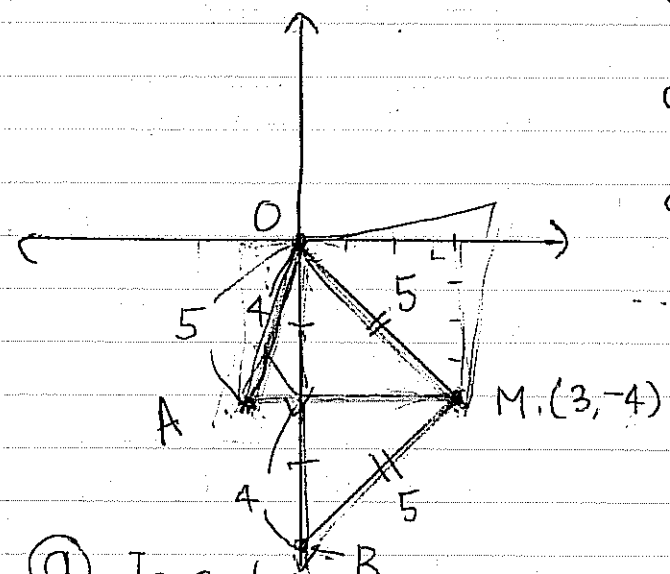
Right Triangle

Vertical

Horizontal

March 11 HW

#8.



$d = \sqrt{3^2 + (-4)^2}$

$d = \sqrt{9 + 16} = \sqrt{25}$

$d = 5$

ⓐ Isosceles

$B = (0, -8)$

ⓑ Equilateral

$d = 5 = \sqrt{x^2 + y^2}$

$5^2 = \left(\sqrt{x^2 + y^2}\right)^2$

$25 = x^2 + y^2$ sub $y = -4$

$25 = x^2 + (-4)^2$

$25 = x^2 + 16$

$25 - 16 = x^2$

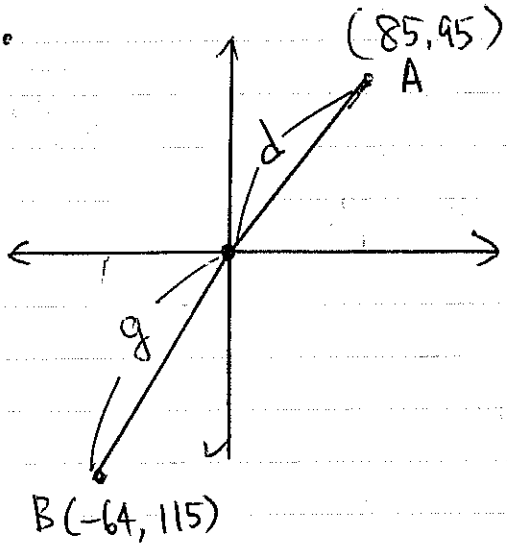
$\sqrt{9} = \sqrt{x^2}$

$\pm 3 = x$

$\therefore$ A is $(-3, -4)$

March 11 HW

#3.



$d = \sqrt{85^2 + 95^2}$

$d = \sqrt{7225 + 9025}$

$d = \sqrt{16250} = 127.5 \text{ km}$

$g = \sqrt{(-64)^2 + (115)^2}$

$g = \sqrt{4096 + 13225}$

$g = \sqrt{17321} = 131.6 \text{ km}$

∴ Plane A should land first.