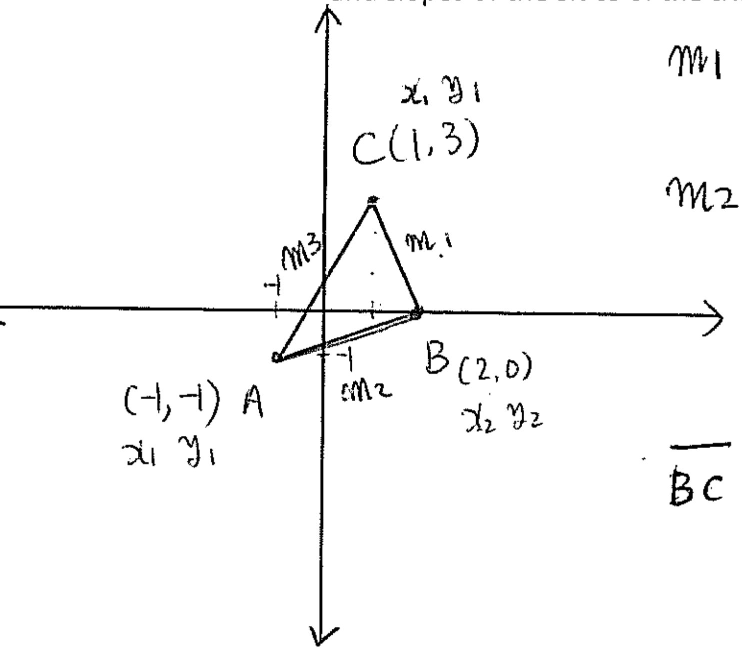
March 12 Lesson * Quiz on Friday

Example 3 A triangle has vertices at A(−1, −1), B(2, 0), and C(1, 3). Find the lengths and slopes of the sides of the triangle. What kind of triangle is it?



$$m_1 = \frac{y_2-y_1}{x_2-x_1} = \frac{0-3}{2-1} = \frac{-3}{1} = -3$$

$$m_2 = \frac{y_2-y_1}{x_2-x_1} = \frac{0--1}{2--1} = \frac{1}{3}$$

$$m_3 = \frac{-1-3}{-1-1} = \frac{-4}{-2} = 2$$

$$\overline{BC} = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
$$= \sqrt{(2-1)^2+(0-3)^2} = \sqrt{1+9}$$
$$= \sqrt{10} = 3.2$$

$$\overline{AB} = \sqrt{(2--1)^2+(0--1)^2}$$
$$= \sqrt{9+1}$$
$$= \sqrt{10} = 3.2$$

$$\overline{AC} = \sqrt{(-1-1)^2+(-1-3)^2}$$
$$= \sqrt{4+16} = \sqrt{20}$$
$$= 4.47$$

∴ This is isoceles right triangle because m_1 is negative reciprocal of m_2. $\overline{AB} = \overline{BC} \rightarrow$ isoceles △

Definitions:

Scalene — Triangle with No equal sides, No equal angles

Isosceles — Two equal ∠s and two equal sides

Equilateral — 3 equal ∠s and 3 equal sides

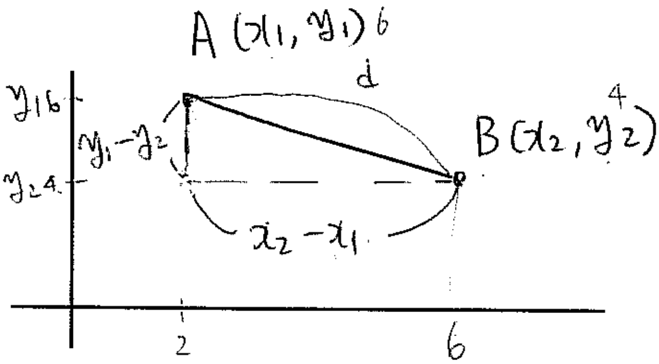
Right Triangle — a triangle with a right angle (90°)

Vertical — Straight up or down → y

Horizontal — Move right or left → x

March 11

The Distance Between Any Two Points



$$d = \sqrt{(y_1 - y_2)^2 + (x_2 - x_1)^2}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

The distance between points $A(x_1, y_1)$ and $B(x_2, y_2)$ in the coordinate plane is

$$\boxed{d = \sqrt{(y_1 - y_2)^2 + (x_2 - x_1)^2}}$$

both are OK.

A **line segment** is the part of a line between two specific points, including the points themselves. The slope of a line segment is the same as the slope of the line containing the line segment.

Parallel line segments have the ___same___ slope.

The slope of two perpendicular line segments are ___negative reciprocal.___.

Example 1 Find the length of the line segments with these end points.

a) $A(-1, 0)$ and $B(5, 2)$
$x_1\ y_1 \quad x_2\ y_2$

$$d = \sqrt{(0-2)^2 + (5 - -1)^2}$$

$$d = \sqrt{4 + 36} = \sqrt{40} = 6.3$$

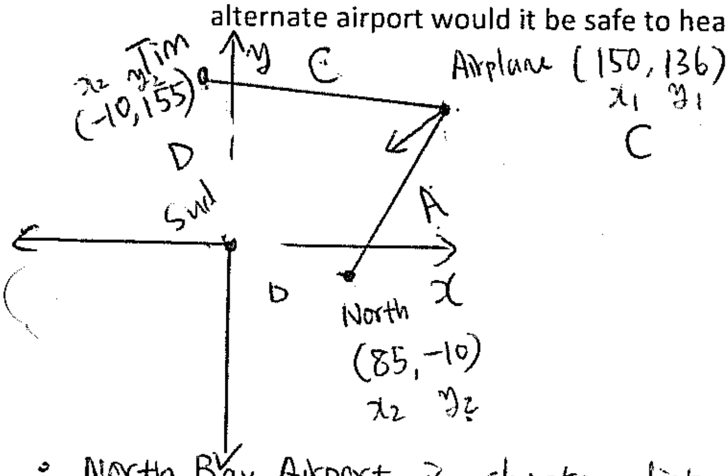
b) $G(-7, 8)$ and $H(-7, -5)$
$x_1\ y_1 \quad x_2\ y_2$

$$d = \sqrt{(8 - -5)^2 + (-7 - -7)^2}$$

$$d = \sqrt{13^2 + 0^2} = \sqrt{13^2}$$

$$\therefore d = 13$$

Example 2 An airplane at coordinates $(150, 136)$, which is heading for Sudbury $(0, 0)$, has to be diverted from poor weather conditions to either North Bay $(85, -10)$ or Timmins $(-10, 155)$. If the airplane is carrying enough fuel to get to Sudbury, which alternate airport would it be safe to head for?



$$C = \sqrt{(-10 - 150)^2 + (155 - 136)^2}$$

$$C = \sqrt{25600 + 361} = \sqrt{25961}$$

$$\therefore C = 161.1$$

$$A = \sqrt{(85 - 150)^2 + (-10 - 136)^2}$$

$$A = \sqrt{4225 + 21316} = 160 \quad \therefore A = 160$$

$\therefore$ North Bay Airport is shorter distance than Timmins

MPM2D

Find the Shortest Distance from a Point to a Line

March 12

$d = \sqrt{(8-6)^2 + (5.5-1.5)^2}$ $(\overline{CR})$

Example 1 Find the Shortest Route

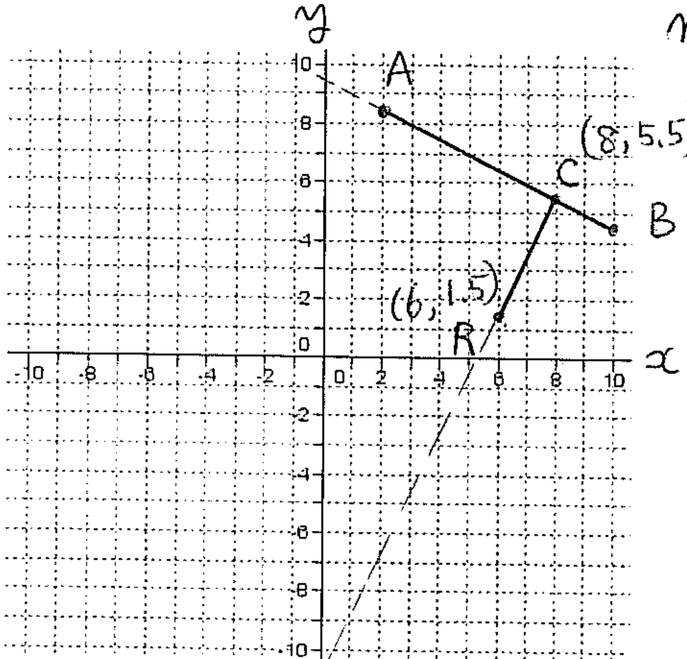
$d = \sqrt{4+16} = \sqrt{20} = 4.47 \times 500m$

$= 2236m$

A ranger cabin is to be built in a flat wooded area near the straight road that connects the two campgrounds in a park. A new side road will connect the cabin to the campground road. On the park map, the campgrounds have coordinates A(2.0, 8.5) and B(10.0, 4.5), while the site for the cabin is at R(6.0, 1.5). Each unit on the map grid represents 500 m.

x_1 y_1 x_2 y_2 x y

a) Find the route that minimizes the cost and the number of trees that have to be cut down for the side road. Draw a diagram of the route.



$m_{AB} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4.5 - 8.5}{10 - 2}$

$= \frac{-4}{8} = -\frac{1}{2}$

$m_{RC} = -\left(-\frac{2}{1}\right) = 2$

$y = 2x + b \leftarrow$ sub R

$1.5 = 2(6) + b$

$1.5 = 12 + b$

$1.5 - 12 = b$

$-10.5 = b$

$\therefore y = 2x - 10.5$

Equation of AB = ? $y = -\frac{1}{2}x + b$

b)

$A = (2.0, 8.5)$

$8.5 = -\frac{1}{2}(2) + b$

$8.5 = -1 + b$

$9.5 = b$

$\therefore \overline{AB} \rightarrow y = -\frac{1}{2}x + 9.5$

Substitution Method:

$2x - 10.5 = -\frac{1}{2}x + 9.5$

$2x + \frac{1}{2}x = 9.5 + 10.5$

$\frac{2}{5} \times \frac{5}{2}x = 20 \times \frac{2}{5}$

$x = 20 \times \frac{2}{5}$

$x = 8 \rightarrow$ Sub int $y = 2x - 10.5$

b) Find the length of the side road.

$y = 2(8) - 10.5$

$y = 16 - 10.5 = 5.5$

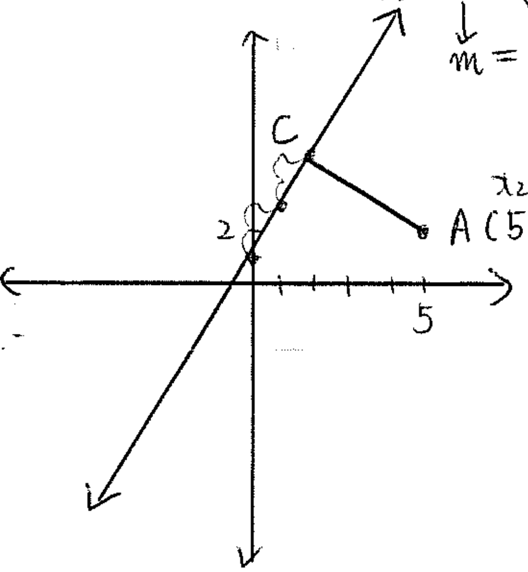
$\therefore C(8, 5.5)$

A

Example 2 Determine the shortest distance from the point (5, 2) to the line represented by $y = 2x + 1$

$= b =$ y intercept $\rightarrow (0, 1)$

$m = \frac{2}{1} = \frac{\text{rise}}{\text{run}}$



$m_{AC} = -\frac{1}{2}$

$\overline{AC} \rightarrow y = -\frac{1}{2}x + b$

$2 = -\frac{1}{2}(5) + b$

$2 = -\frac{5}{2} + b$

$2 + \frac{5}{2} = b$

$\frac{4+5}{2} = b$

$\frac{9}{2} = b$

$\therefore \overline{AC}$

$y = -\frac{1}{2}x + \frac{9}{2}$

* Substitution Method

$2x + 1 = -\frac{1}{2}x + \frac{9}{2}$

$2x + \frac{1}{2}x = \frac{9}{2} - 1$

$\frac{4x + x}{2} = \frac{9-2}{2}$

$2 \times \frac{5x}{2} = \frac{7}{2} \times 2$

$5x = 7$

$x = \frac{7}{5}$

* sub $x = \frac{7}{5}$ into $y = 2x + 1$

$y = 2\left(\frac{7}{5}\right) + 1 = \frac{14}{5} + 1 = \frac{14}{5} + \frac{5}{5}$

$y = \frac{19}{5}$ $\therefore$ POI $= \left(\frac{7}{5}, \frac{19}{5}\right) \Rightarrow C$

x_1 y_1

D of $\overline{AC} = \sqrt{\left(\frac{5}{1} - \frac{7}{5}\right)^2 + \left(2 - \frac{19}{5}\right)^2}$

$D = \sqrt{\left(\frac{25}{5} - \frac{7}{5}\right)^2 + \left(\frac{10}{5} - \frac{19}{5}\right)^2} = \sqrt{\left(\frac{18}{5}\right)^2 + \left(\frac{-9}{5}\right)^2}$

$= \sqrt{12.96 + 3.24}$

$= \sqrt{16.2}$

$= 4.0$

1. Determine the shortest distance from the point D(5, 4) to the line represented by $3x + 5y - 4 = 0$
2. A cable company is connecting a new customer to its cable network. On a site plan, the customer's house has coordinates H(7, 17). The equation $y = \frac{1}{2}x + 4$ represents the existing trunk cable. The cable company wants to keep the branch to the customer's house as short as possible.
   a. Where should the cable company make the connection to the trunk cable?
   b. How long will the branch connection be if each unit on the grid of the site plan represents 10 m?

$\therefore$ The shortest distance is 4.

Answers: 1. 5.32 m 2. a) (10.8, 9.4) b) 85 m