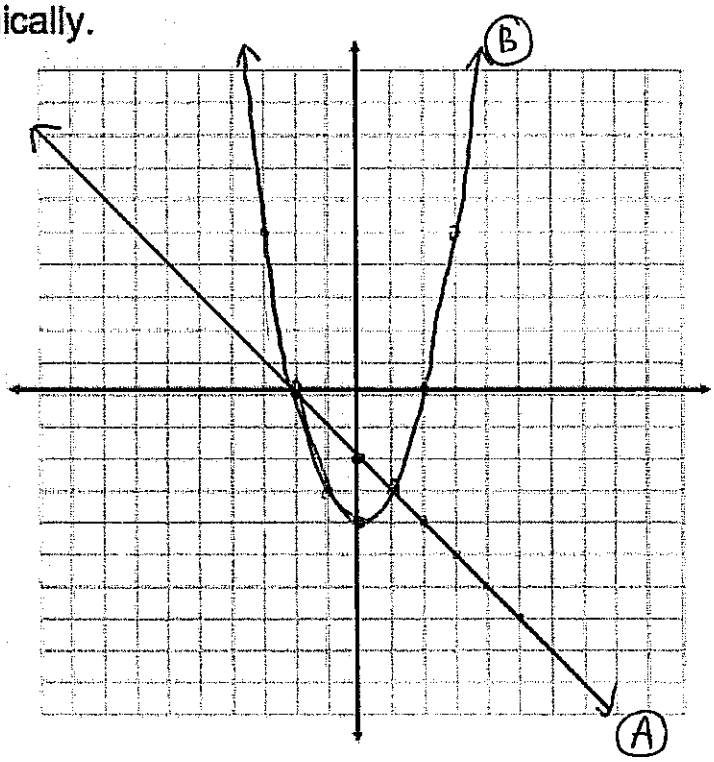
March 31 Park

Linear-Quadratic Systems

1) Solve the system graphically.

$y = -x - 2$ — (A)

$y = x^2 - 4$ — (B)



These two graphs intersect at $(1, -3)$ and $(-2, 0)$.

or two solutions are $(1, -3)$ and $(-2, 0)$

2) Solve algebraically.

$y = 3x - 3$ — (A)

$y = x^2 - 4x + 7$ — (B)

sub (A) into (B)

$3x - 3 = x^2 - 4x + 7$

$0 = x^2 - 4x - 3x + 3 + 7$

$0 = x^2 - 7x + 10$

$0 = (x-5)(x-2)$

$x = 5, 2$

$ac = 10$
$b = -7$
$-5 \times -2 = 10$

* Sub $x = 5$ into (A)

$y = 3(5) - 3 = 15 - 3 = 12$

When $x = 5$, $y = 12$

* Sub $x = 2$ into (A)

$y = 3(2) - 3 = 3$

when $x = 2$, $y = 3$

∴ Two POI are $(5, 12)$ and $(2, 3)$

3) Determine the number of points of intersection for each system.

a) $y = 3x + 5$ — Ⓐ

$y = 3x^2 - 2x - 4$ — Ⓑ

Sub Ⓐ into Ⓑ

$3x + 5 = 3x^2 - 2x - 4$

$0 = 3x^2 - 2x - 3x - 4 - 5$

$0 = 3x^2 - 5x - 9 \rightarrow ax^2 + bx + c$

Discriminant: $b^2 - 4ac$

$= (-5)^2 - 4(3)(-9)$

$= 25 + 108 = 133 > 0$

∴ There are 2 solutions or two POI.

b) $y = -x - 2$ — Ⓐ

$y = -2x^2 + x - 3$ — Ⓑ

Sub Ⓐ into Ⓑ

$-x - 2 = -2x^2 + x - 3$

$0 = -2x^2 + x + x - 3 + 2$

$0 = -2x^2 + 2x - 1$

$b^2 - 4ac = 2^2 - 4(-2)(-1)$

$= 4 - 8 = -4 < 0$

∴ Since the discriminant is less than zero, there are no solutions or no POI.

Summary

* A linear function and a quadratic function may:

① intersect at two points

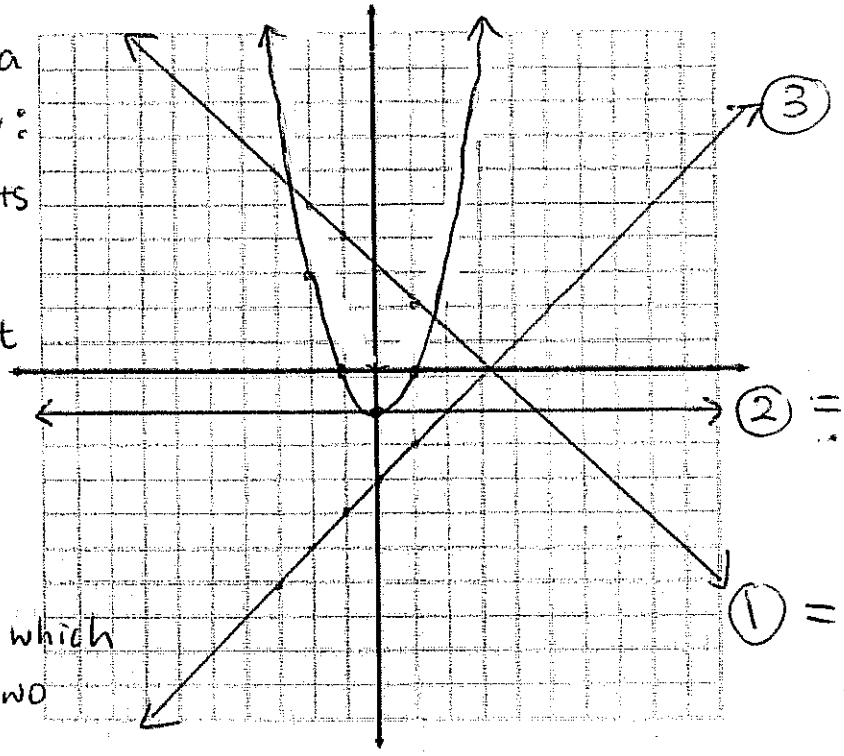
$\Rightarrow b^2 - 4ac > 0$

② intersect at one point

$\Rightarrow b^2 - 4ac = 0$

③ Never intersect

$\Rightarrow b^2 - 4ac < 0$



* secant line — is a line which intersects a curve at two points.

* Tangent line — is a line which touches a curve at one point and has the slope of the curve at that point.

Hwk. pg. 67 # 1, 3, 7, 10