April 1st Park

MCR3U

Linear-Quadratic Systems

$y = ax^2 + bx + c$

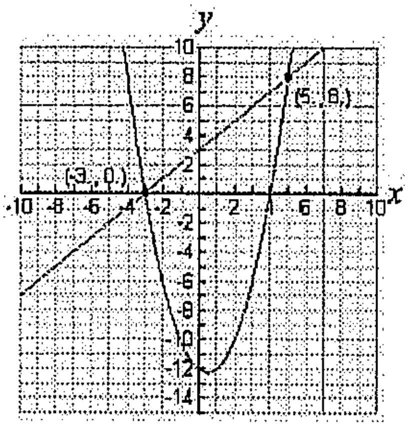
A **linear-quadratic system** contains a parabola and a line with the same set of variables.

$y = mx + b$

The **solution to a linear-quadratic system** is the intersection point of a line and a parabola.

or POI

There are 3 ways a line and a parabola can intersect:

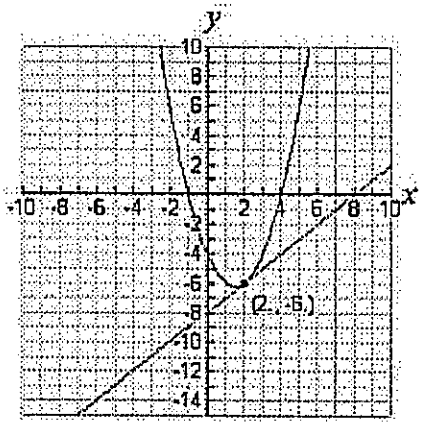


$b^2 - 4ac > 0$

two solutions (= POI)

The line may be called a

secant

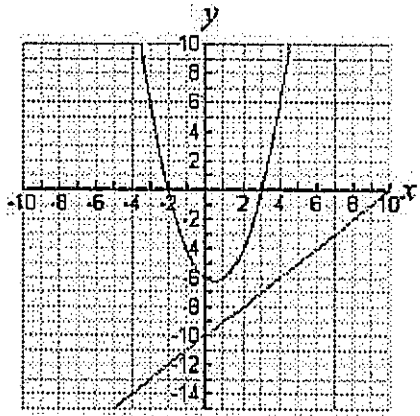


$b^2 - 4ac = 0$

one solution (= POI)

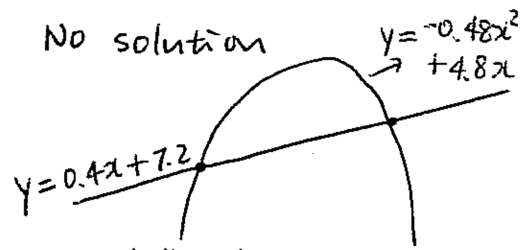
The line may be called a

tangent



$b^2 - 4ac < 0$

No solution



$y = 0.4x + 7.2$ (A)

Example 1 Maria is a set designer. In one scene, a banner will hang across a parabolic archway. To make it look interesting, she will put the banner on an angle. She sets the banner along a line defined by the linear equation: where x represents the horizontal distance and y represents the vertical destance in metres, from one foot of the archway. The archway is modelled by the quadratic equation: $y = -0.48x^2 + 4.8x$ (B)

a) What points along the archway should she attach the banner?

Let x represent horizontal distance

" y " vertical distance

* Sub (A) into (B)

$0.4x + 7.2 = -0.48x^2 + 4.8x$

$0 = -0.48x^2 + 4.4x - 7.2$

Divide LS and RS by -0.48

$0 = x^2 - 9.17x + 15$

$QF = \dfrac{9.17 \pm \sqrt{(9.17)^2 - 4(1)(15)}}{2}$

$QF = \dfrac{9.17 \pm 4.91}{2} = 7, 2.13$

* Sub $x = 7 \rightarrow$ (A) $y = 0.4(7) + 7.2$

$= 10$

$\therefore (7, 10)$

* Sub $x = 2.13 \rightarrow$ (A) $y = 0.4(2.13) + 7.2$

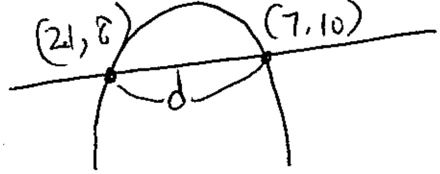
$= 8$

$\therefore (2.1, 8)$

$\therefore$ She should attach the banner at (7, 10) and (2.1, 8)

$(7, 10)$ $(2.1, 8)$

x_2, y_2 x_1, y_1



b) What is the length of the banner?

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(7 - 2.1)^2 + (10 - 8)^2} = \sqrt{4.9^2 + 2^2} = \sqrt{28.0} = 5.29$$

$$= 5.3 \text{ m}$$

Example 2 Using the discriminant to determine the number of intersection points

1. Determine the number of points of intersection of the quadratic and linear functions $f(x) = 3x^2 + 12x + 14$ and $g(x) = 2x - 8$.

$$3x^2 + 12x + 14 = 2x - 8$$

$$3x^2 + 10x + 22 = 0$$

$$b^2 - 4ac = 10^2 - 4(3)(22)$$

$$= -164 < 0$$

$\therefore$ There are no points of intersection

2. Determine the value of k such that $g(x) = 3x + k$ intersects the quadratic function $f(x) = 2x^2 - 5x + 3$ at exactly one point. $\longrightarrow b^2 - 4ac = 0$

$$3x + k = 2x^2 - 5x + 3$$

$$0 = 2x^2 - 5x - 3x + 3 - k$$

$$0 = \underset{a}{2}x^2 \underset{b}{-8}x + \underset{c}{(3 - k)}$$

$$b^2 - 4ac = 0$$

$$(-8)^2 - 4(2)(3 - k) = 0$$

$$64 - 8(3 - k) = 0$$

$$64 - 24 + 8k = 0$$

$$\frac{8k}{8} = \frac{-40}{8}$$

$\therefore k = -5$

3. A quadratic function is defined by $f(x) = 3x^2 + 4x - 2$. A linear function is defined by $g(x) = mx - 5$. What value(s) of the slope of the line would make it a tangent to the parabola?

$b^2 - 4ac = 0$

$$mx - 5 = 3x^2 + 4x - 2$$

$$0 = 3x^2 + 4x - mx - 2 + 5$$

$$0 = \underset{a}{3}x^2 + \underset{b}{(4 - m)}x + \underset{c}{3}$$

$$b^2 - 4ac = 0$$

$$(4 - m)^2 - 4(3)(3) = 0$$

$$16 + m^2 - 8m - 36 = 0$$

$$m^2 - 8m - 20 = 0$$

$$(m - 10)(m + 2) = 0$$

$ac = -20$
$b = -8$
$-10 \times 2 = -20$

$\therefore m = 10, -2$

$\therefore$ Slope of 10 and -2 would make it a tangent line.

HW pg 67 # 5a, c

(yesterday's) # 7, 10