Verifying Geometric Properties

MPM2D

Friday: Unit 3 test!

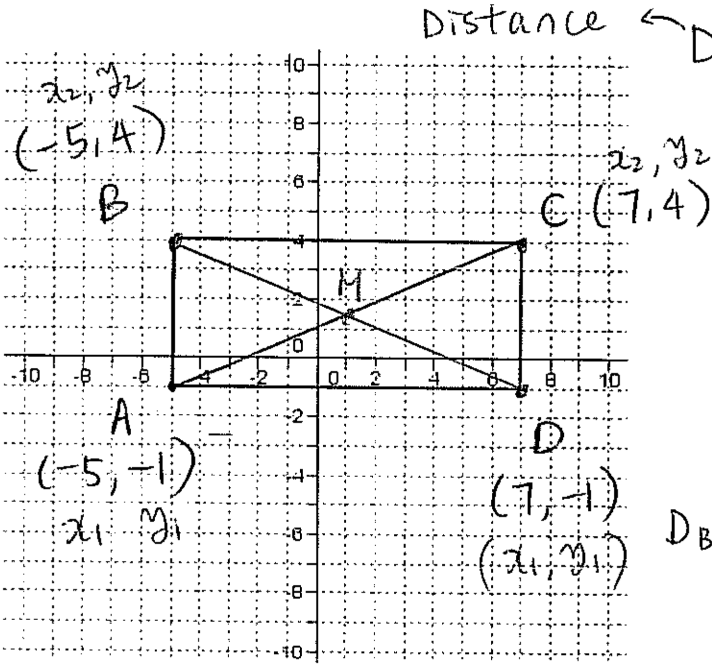
Recall:

Diagonal – a line drawn from one vertex to another vertex through the shape

Bisect – splits a line segment in two equal parts.

1. **Rectangle** at coordinates A (−5, − 1), B (−5, 4), C (7, 4) D (7, − 1)

Verify that the diagonals of this rectangle are equal in length and bisect each other.



Distance ← $D_{AC} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(7-(-5))^2 + (4-(-1))^2}$

$= \sqrt{12^2 + 5^2} = \sqrt{144+25}$

$= 13$ units

$D_{BD} = \sqrt{(-5-7)^2 + (4-(-1))^2}$

$= \sqrt{144+25} = 13$

$\therefore D_{AC} = D_{BD}$

∴ The Diagnolls are equal in length.

"Bisect each other" → Midpoint of AC and BD

$M_{AC} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$M_{AC} = \left(\frac{-5+7}{2}, \frac{-1+4}{2}\right)$

$M_{AC} = (1, 1.5)$

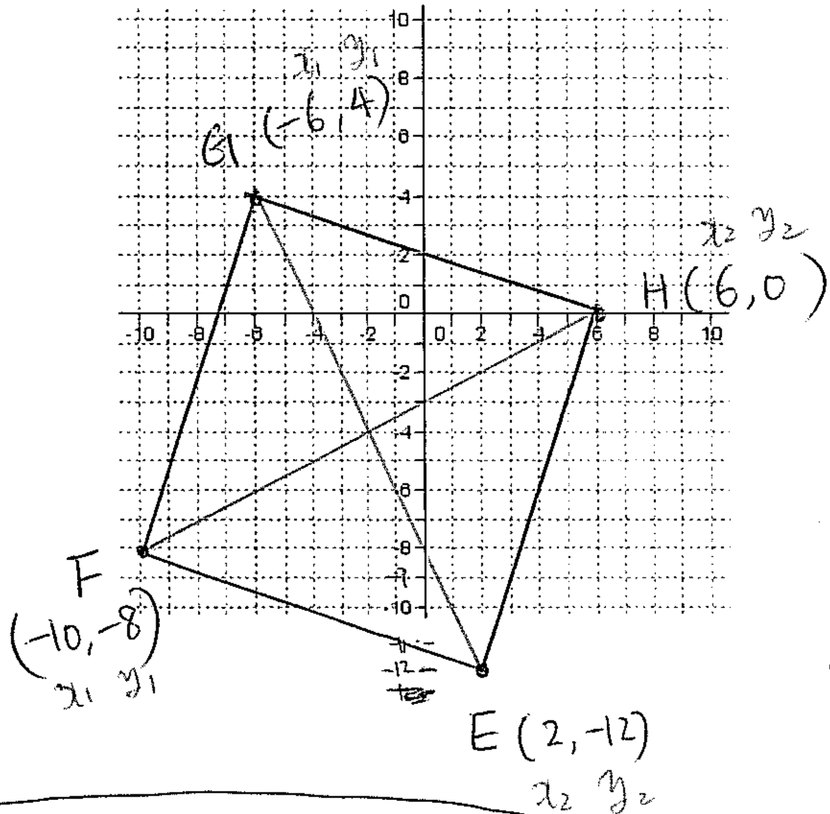
$M_{BD} = \left(\frac{-5+7}{2}, \frac{4+(-1)}{2}\right)$

$M_{BD} = (1, 1.5)$

∴ Since the midpoints are the same, the diagnals bisect each other.

2. **Square** at coordinates E (2 , −12), F (−10 , − 8), G (−6, 4), H (6 , 0)

Verify that the diagonals of this square are equal in length, bisect each other and are perpendicular.



"Equal in length"

$$D_{GE} = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$

$$D_{GE} = \sqrt{(2--6)^2 + (-12-4)^2}$$

$$D_{GE} = \sqrt{8^2 + (-16)^2}$$

$$D_{GE} = \sqrt{320} = 17.89$$

$$D_{FH} = \sqrt{(6--10)^2 + (0--8)^2}$$

$$D_{FH} = \sqrt{16^2 + 8^2} = 17.89$$

∴ The diagnols are equal in length.

"Bisector"

$$\overset{\text{Midpoint}}{M}_{GE} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$M_{GE} = \left(\frac{-6+2}{2}, \frac{4-12}{2}\right)$$

$$M_{GE} = \left(\frac{-4}{2}, \frac{-8}{2}\right) = (-2, -4)$$

$$M_{FH} = \left(\frac{-10+6}{2}, \frac{-8+0}{2}\right) = (-2, -4)$$

Since $M_{GE} = M_{FH}$, they bisect each other.

"Perpendicular"

$$\overset{\text{slope}}{M}_{GE} = \frac{y_2-y_1}{x_2-x_1}$$

$$= \frac{-12-4}{2--6} = \frac{-16}{8}$$

$$M_{GE} = -2$$

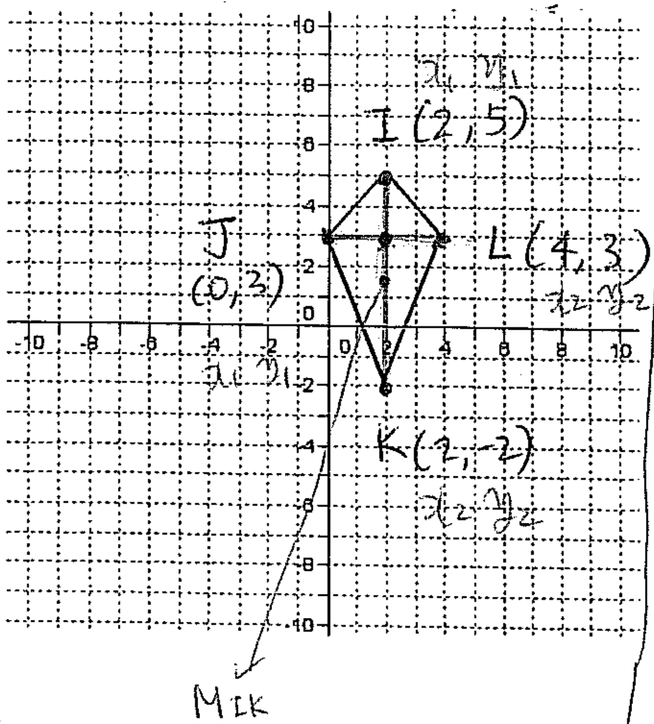
$$M_{FH} = \frac{0--8}{6--10} = \frac{8}{16}$$

$$= \frac{1}{2}$$

∴ Since these two slopes are negative reciprocal, they are perpendicular to each other.

3. **Kite** at coordinates I (2,5), J (0,3) , K (2,−2) , L (4,3).

Verify that this kite is a shape in which one of the diagonals is the perpendicular bisector of the other.



"Perpendicular"

$$m_{IK} = \frac{y_2 - y_1}{x_2 - x_1}$$

slope

$$= \frac{-2-5}{2-2} = \frac{-7}{0} = \text{undefin.}$$

$$m_{JL} = \frac{3-3}{4-0} = \frac{0}{4} = 0$$

Since M_{IK} is vertical line and M_{JL} is horizontal line, they are perpendicular to each other.

"Bisector"

Midpoint

$$M_{IK} = \left(\frac{2+2}{2}, \frac{5-2}{2}\right) = \left(\frac{4}{2}, \frac{3}{2}\right) = (2, 1.5)$$

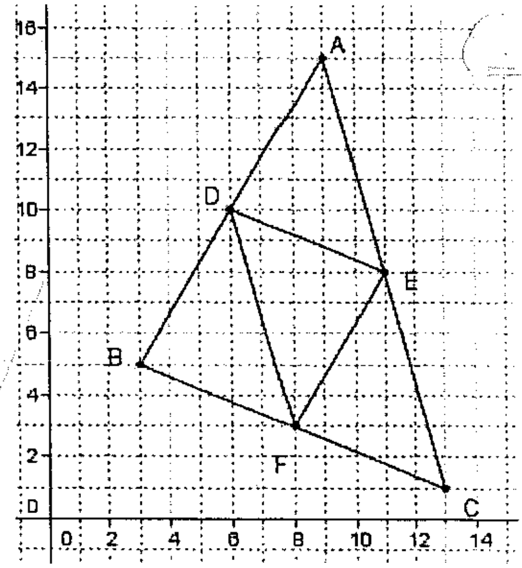
$$M_{JL} = \left(\frac{0+4}{2}, \frac{3+3}{2}\right) = (2,3)$$

$\therefore$ Since M_{JL} is a midpoint of $\overline{JL}$, $\overline{IK}$ is a perpendicular and it bisects $\overline{JL}$.

Homework: Worksheet

Verifying Geometric Properties Worksheet

1.



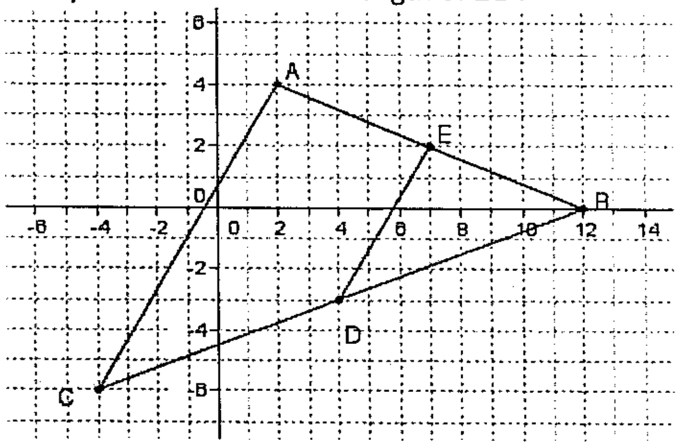
a) Verify that DE and BC are parallel.

b) List the other line segments that are parallel.

c) Verify that $DE = BF$.

d) List the other line segments that have equal lengths.

2. Verify that AC is twice the length of ED.



3.

a) Draw the quadrilateral with vertices $P(0,7)$, $Q(-2,1)$, $R(4,-1)$, and $S(6,3)$.
b) Find the midpoint of each side. Join the midpoints of the adjacent sides to form a new quadrilateral $TUVW$.
c) Verify that opposite sides of $TUVW$ are parallel.
d) Verify that opposite sides of $TUVW$ are equal in length.
e) What shape is $TUVW$?

4.

a) Draw the trapezoid with vertices $A(-2,-2)$, $B(2,-2)$, $C(4,1)$, and $D(2,4)$.
b) Verify that the line segment joining the midpoints of the non-parallel sides of the trapezoid is parallel to the other two sides.

5.

a) Draw the rhombus with vertices $A(-5,2)$, $B(1,3)$, $C(-2,-1)$, and $D(-6,-2)$.
b) Verify that joining the midpoints of the adjacent sides of $ABCD$ produces a rectangle.