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c) A town with a population of 50 000 has been decreasing at an average rate of 3.1% for the last 10 years. Find an equation for the situation for population in the thousands.

$Y = 50\,(0.969)^x$

* 3.1% = 0.031
* 1 − 0.031 = 0.969
* Let y be the population in thousa
* // x // // # of years.

d) A computer loses 5% of its value each month after it is purchased. The original price was $1 200.

$Y = 1200\,(0.95)^x$

* 5% = 0.05
* 1 − 0.05 = 0.95
* Let y

e) In 1990, a sum of $10 000 is invested at a rate of 3% per year.

$y = 10000\,(1.03)^t$

Let y be the amount of money.
Let t // time in years.

How are these questions different from the doubling and half-life questions?

radioactive element from Superman's planet

↳ "b" is either 1 − x% or 1 + x%

Create one doubling or half-life question, and find the equation that models it.

Kryptonite has a half life of 10 months. You have 5000 g. How much Kryptonite will be left after 55 months?

Let y be the mass of Kryptonite in g.
// x // time in months.

$Y = 5000\left(\frac{1}{2}\right)^{\frac{t}{10}}$ → Equation

$Y = 5000\left(\frac{1}{2}\right)^{\frac{55}{10}}$ ← sub t = 55

$Y = 5000\,(0.5)^{5.5} = 110.49$ g

Create one appreciation/depreciation question, and find the equation that models it.

Homework: Pg. 154 #10-12

optional → Physics interest: pg. 168 #18-20

Pg. 168 #12, 16, 17
Thinking pg. 169#21, 22

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MCR3U
Ms. Kueh

Exponential Growth and Decay

Knowledge

1. The population of a city is 810 000. If it is increasing by 4% per year, estimate the population in four years.

2. A painting, purchased for $10 000 in 1990, increased in value by 8% per year. Find the value of the painting in the year 2000.

3. Inflation is causing things to cost roughly 2% more per year. A bag of milk cost $3.75 now. Estimate its cost in five years.

4. The world population is doubling about every 35 years. In 1980 the total population was about 4.5 billion. If the doubling period remains at 35, find the projected world population for the year 2120.

5. An isotope of sodium, Na-24 has a half-life of 15 h. Find the amount remaining from a 4 g sample after 3 days.

6. A used car dealer sells a five-year-old car for $4200. What was the original value of the car if the depreciation is 15% a year?

Application and Thinking

7. A bacteria culture starts with 3000 bacteria. After 3 hours the estimated count is 48 000. What is the doubling period?

8. A research assistant made 160 mg $= a$ of radioactive sodium, Na-24, and found that there was only 20 mg left 45 h later. $\rightarrow$ (45, 20)
   a) What is the half-life of Na-24?
   b) Find an equation that models the amount, A, after *t* hours.
   c) If the laboratory requires 100 mg of Na-24 12 h from now, how much Na-24 should the research assistant make now? (Ignore the 20 mg she currently has.)
   d) How much of the original 20 mg would be left in 12 h?

9. Two different strains of cold virus were isolated and put in cultures to grow. Virus A triples every 8 h while virus B doubles every 4.8 h. If each culture has 1000 viruses to start, which has more after 24 h?

10. A bacteria colony grows at the rate of 15%/h. $\Rightarrow b = 1.15^x$
    a) In how many hours will the colony double in size? (What must you do if you can't get the same base?) When $y = 2y \rightarrow x = ?$
    b) In 10 h the bacteria population grows to 1.3×10^3. How many bacteria were there initially?

11. The biological half-life of thyroid hormone T4 is about 6.5 days. If a dose of T4 was not followed by repeat doses,
    a) What fraction of the original dose would remain in the body after 19.5 days?

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(Text)

P196 # 8 $y = 3^{-0.5(x+2)} - 5$

$y = 3^{-0.5x-1} - 5$

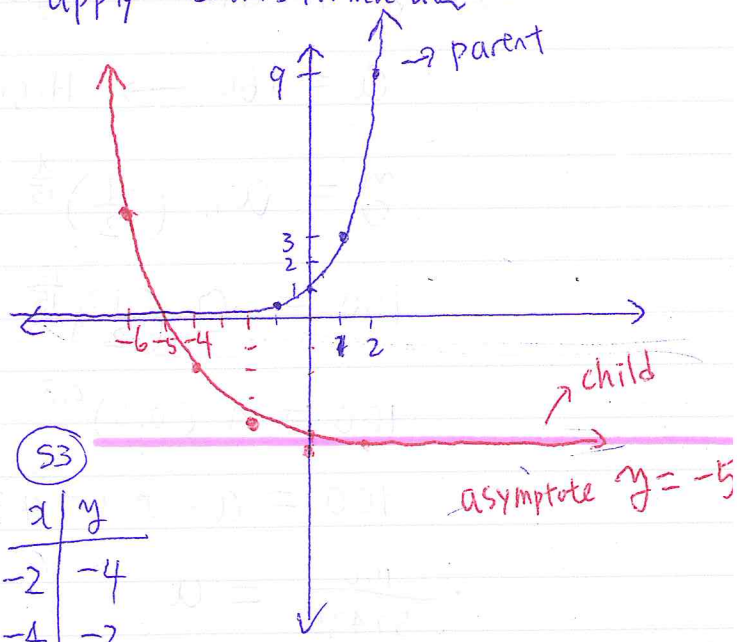
Use $y = 3^x$ as the base and apply transformation

Step 1: Reflect on y axis

Step 2: Horizontal Stretch by @ 2

Step 3: Shift to the left by 2

and down by 5



parent = original

x	y
0	1
1	3
2	9
-1	$\frac{1}{3}$
-2	$\frac{1}{9}$

(S1)

x	y
0	1
-1	3
-2	9
1	$\frac{1}{3}$
2	$\frac{1}{9}$

(S2)

x	y
0	1
-2	3
-4	9
2	$\frac{1}{3}$
4	$\frac{1}{9}$

(S3)

x	y
-2	-4
-4	-2
-6	4
0	-4.67
2	-4.88

#8 a) "Exponential Growth and Decay" worksheet

[Let x represent time in hours.
[Let y // mass of RS in mg.

$a = 160$ $b = 0.5$

$y = 160(0.5)^{\frac{x}{h}}$

sub (45, 20)

$\frac{20}{160} = \frac{160(0.5)^{\frac{45}{h}}}{160}$

$\frac{1}{8} = 0.5^{\frac{45}{h}}$

$\frac{1}{8} = \left(\frac{1}{2}\right)^{\frac{45}{h}}$

$\downarrow = \frac{1^3}{2^3} = \left(\frac{1}{2}\right)^3$

$\left(\frac{1}{2}\right)^3 = \left(\frac{1}{2}\right)^{\frac{45}{h}}$

$3 = \frac{45}{h}$

$\frac{3h}{3} = \frac{45}{3}$

$h = 15$

∴ Half life is 15 hours

#8b) $y = 160\left(\frac{1}{2}\right)^{\frac{x}{15}}$

c) $x = 12$ hours $\rightarrow$ $y = 100$ mg

$a = 160 \rightarrow$ Higher #

$y = a_n\left(\frac{1}{2}\right)^{\frac{x}{15}}$ ← sub $(12, 100)$

$100 = a\left(\frac{1}{2}\right)^{\frac{12}{15}}$

$100 = a\,(0.5)^{0.8}$

$100 = a \cdot 0.57435$

$\frac{100}{0.57435} = a$

$\therefore a = 174$ mg

$\therefore$ They should make 174 mg of Sodium Na 24

d) Question: If initial amt was 20 mg, how much will still be left after 12 hours?

$y = 20\left(\frac{1}{2}\right)^{\frac{x}{15}}$ ← Sub $x = 12$ hours

$y = 20\left(\frac{1}{2}\right)^{\frac{12}{15}}$

$y = 20\,(0.5)^{0.8}$

$y = 11.5$ mg

#6. $y = a\,(0.85)^{x}$ ← When $x = 5$ years, $y = 4200$

Sub $(5, 4200)$

$4200 = a\,(0.85)^{5}$

$4200 = a \cdot 0.4437$

$\frac{4200}{0.4437} = a \rightarrow \therefore a = 9465.86$

[Let x = time in years
Let y = amount of car's value

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#9 "Exponent Growth"

Virus A: $b = 3^{\frac{x}{8}}$ $\quad a = 1000$

Virus B: $b = 2^{\frac{x}{4.8}}$ $\quad a = 1000$

When $x = 24$ hours → which y is bigger?

Virus A: $y = 1000(3)^{\frac{x}{8}}$ ← sub $x = 24$

$y = 1000(3)^{\frac{24}{8}}$

$= 27000$ viruses A

Virus B: $y = 1000(2)^{\frac{x}{4.8}}$ ← sub $x = 24$

$y = 1000 \cdot (2)^5$

$= 32000$ virus B

∴ Virus B type has more viruses after 24 hours.

#10. a) $y = a(1.15)^x$

$\frac{2a}{a} = \frac{a(1.15)^x}{a}$ ← sub $2a$ for y

$2 = 1.15^x$ → By using calculator, you try many #. Then you can find $1.15^5 = 2.01$ ∴ $x = 5$ hours

b) When $x = 10$ hours → $y = 1.3 \times 10^3$

$1.3 \times 10^3 = 1300 = y$

$1300 = a(1.15)^{10}$ → $a = \frac{1300}{1.15^{10}}$

$a = 321.34$