April 10 = Fractional Exponents

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Mr. Park

Rational Exponents

Recall: Evaluate **without** a calculator. Youtube: "Rational Exponents" by Bill Witte

$2^2 = 4$ $3^2 = 9$ $5^2 = 25$ $4^2 =$ $4^3 =$

$2^4 = 16$ $3^3 = 27$ $5^3 = 125$ $2^5 =$ $6^2 =$

Investigate the meaning of a rational exponent:

1. a) What operations can we perform to change $16^{\frac{1}{2}}$ into 16?

$16^{\frac{1}{2}} \times 16^{\frac{1}{2}} = 16^{\frac{1}{2}+\frac{1}{2}} = 16^1 = 16$

$(16^{\frac{1}{2}})^2 = 16^{\frac{1}{2}\times 2} = 16^1 = 16$

b) What does $16^{\frac{1}{2}}$ mean?

$16^{\frac{1}{2}} = \sqrt{16}$

because $(\sqrt{16})^2 = 16$ and $(16^{\frac{1}{2}})^2 = 16$

2. a) What operations can we perform to change $8^{\frac{1}{3}}$ into 8?

$(8^{\frac{1}{3}})^3 = 8^{\frac{1}{3}\times 3} = 8^1 = 8$

a) What does $8^{\frac{1}{3}}$ mean?

Let $x = 8^{\frac{1}{3}} \rightarrow (x)^3 = (8^{\frac{1}{3}})^3 \rightarrow x^3 = 8$

$\therefore x = \sqrt[3]{8}$

3. a) What operations can we perform to change $16^{\frac{1}{4}}$ into 16?

$(16^{\frac{1}{4}})^4 = 16^{\frac{1}{4}\times 4} = 16^1 = 16$

b) What does $16^{\frac{1}{4}}$ mean? Let $x = 16^{\frac{1}{4}}$

$16^{\frac{1}{4}} = \sqrt[4]{16}$

$x^4 = (16^{\frac{1}{4}})^4$

$x^4 = 16^{\frac{1}{4}\times 4}$

$\sqrt[4]{x^4} = \sqrt[4]{16^1}$

$\sqrt[4]{x} = \sqrt[4]{16}$

$\sqrt[4]{x} = \sqrt[4]{2^4}$

$x^{\frac{1}{4}} = 2$

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4. What you do think $27^{\frac{1}{3}}$ would be? $27^{\frac{1}{3}} = \sqrt[3]{27}$

5. What you do think $32^{\frac{1}{5}}$ would be? $32^{\frac{1}{5}} = \sqrt[5]{32}$

* Denominator of the fractional exponent is index of the radical number.

Therefore $\boxed{a^{\frac{1}{n}} = \sqrt[n]{a}}$

1. Rational exponents of form $\frac{1}{n}$

a. $\left(\frac{16}{81}\right)^{\frac{1}{4}} = \sqrt[4]{\frac{16}{81}} = \frac{\sqrt[4]{16}}{\sqrt[4]{81}} = \frac{\sqrt[4]{2^4}}{\sqrt[4]{3^4}} = \frac{2^{\frac{1}{4}\times 4}}{3^{\frac{1}{4}\times 4}} = \frac{2}{3}$

b. $(-32)^{\frac{1}{5}} = \sqrt[5]{-32} = \sqrt[5]{(-2)^5} = (-2)^{5\times\frac{1}{5}} = -2$

c. $(-16)^{\frac{1}{4}} = \sqrt[4]{-16} \neq \sqrt[4]{(-2)^4}$ ∴ can't do any more

d. $(-27)^{-\frac{1}{3}}$

$= \frac{1}{(-27)^{\frac{1}{3}}} = \frac{1}{\sqrt[3]{-27}} = \frac{1}{(-3)^{3\times\frac{1}{3}}} = \frac{1}{-3}$

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* Note: $(-2)(-2)(-2)(-2)(-2) = -32$
$(-2)(-2)(-2)(-2) = 16$

about (b) and (c)

You can take out negative base number (which is inside of a radical) only if n (index) is odd number.

2. Solve.

a. $x^2 = -16$ $\sqrt{x^2} = \sqrt{-16}$ → Not solvable

b. $\sqrt[3]{x^3} = \sqrt[3]{-64}$ $\sqrt[3]{x^3} = \sqrt[3]{(-4)^3} \Rightarrow x = -4$

c. $\sqrt[4]{x^4} = \sqrt[4]{10000}$ $\sqrt[4]{x^4} = \sqrt[4]{10^4} \Rightarrow x = 10$

3. Rational Exponents of the form $\frac{m}{n}$

a. $8^{\frac{2}{3}} = \sqrt[3]{8^2} = \sqrt[3]{(2^3)^2} = \sqrt[3]{2^6} = 2^{6\times\frac{1}{3}} = 2^2 = 4$

b. $81^{\frac{5}{4}} = \sqrt[4]{(3^4)^5} = 3^{4\times 5\times\frac{1}{4}} = 3^5 = 243$

c. $\left(\frac{49}{81}\right)^{-\frac{3}{2}} = \frac{1}{\left(\frac{49}{81}\right)^{\frac{3}{2}}} = \frac{1}{\left(\frac{7^{2\times\frac{3}{2}}}{9^{2\times\frac{3}{2}}}\right)} = \frac{1}{\frac{7^3}{9^3}}$

$\frac{9^3}{7^3} = \frac{729}{343}$

$\frac{1}{\frac{\sqrt[2]{(7^2)^3}}{\sqrt[2]{(9^2)^3}}} = \frac{1}{\frac{7^3}{9^3}}$

Also you can use this method

Therefore $\boxed{a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m}$

* If n is even, a must be greater than or equal to zero.

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4. Simplify. Express your answer using only positive exponents.

a. $\dfrac{x^{\frac{2}{3}}x^{\frac{2}{3}}}{x^{\frac{1}{3}}} = \dfrac{x^{\frac{2}{3}+\frac{2}{3}}}{x^{\frac{1}{3}}} = \dfrac{x^{\frac{4}{3}}}{x^{\frac{1}{3}}} = x^{\frac{4}{3}-\frac{1}{3}} = x^{\frac{3}{3}} = x^1 = x$

b. $\left(5x^{\frac{1}{2}}\right)^2\left(4x^{-\frac{1}{2}}\right)$

$x = (5x)^{\frac{1}{2}\times 2}\left(\dfrac{1}{4x^{\frac{1}{2}}}\right) = \dfrac{5x}{4x^{\frac{1}{2}}} = \dfrac{5}{4}x^{1-\frac{1}{2}} = \dfrac{5}{4}x^{\frac{1}{2}}$

5. Express $\sqrt{5}$ as an expression with exponents.

$\sqrt{5} = 5^{\frac{1}{2}}$

6. Express $\sqrt[6]{7}$ as an expression with exponents.

$\sqrt[6]{7} = 7^{\frac{1}{6}}$

7. Express $\sqrt[4]{10000m^8}$ as an expression with exponents, then evaluate.

$\sqrt[4]{10000\,m^8} = \sqrt[4]{10^4 \cdot m^8} = \sqrt[4]{10^4} \cdot \sqrt[4]{m^8} = 10 \cdot m^{8\times\frac{1}{4}}$

$= 10m^2$

Homework: pg. 175 #1-8, 12