April 1

MPM2D
Ms. Kueh

Similar and Congruent Triangles

Two triangles may be congruent, similar, or neither. The order of the letters naming the vertices of congruent or similar triangles indicates the order in which the vertices, sides, and angles of one triangle correspond to the vertices, sides, and angles of the other.

Congruent

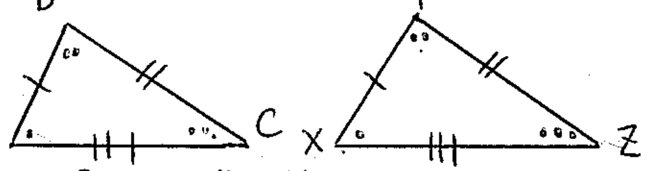
Definition: Twins

Congruent Δs are identical in every way. They have exactly the same shape and size.
The expression:

$\triangle ABC \cong \triangle XYZ$

means that ΔABC is congruent to ΔXYZ

If $\Delta ABC \cong \Delta XYZ$, then



Corresponding sides are equal

$AB = XY$ $\quad AC = XZ$

$BC = YZ$

Corresponding angles are equal.

$\angle ABC = \angle XYZ$ (··)

$\angle BAC = \angle YXZ$ (·)

$\angle ACB = \angle XZY$ (···)

Similar

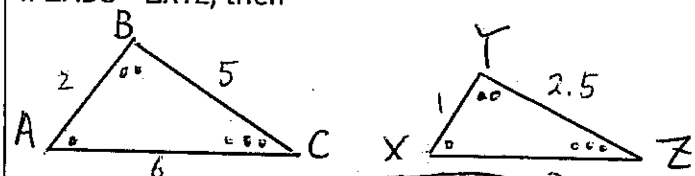
Definition: Father and son

Similar Δs have the same shape, but are different sizes. One triangle is an larger or smaller than the other.
The expression:

$\triangle ABC \sim \triangle XYZ$

means that ΔABC is similar to ΔXYZ

If $\Delta ABC \sim \Delta XYZ$, then



Corresponding sides are proportional

$$\frac{AB}{XY} = \frac{BC}{YZ} = \frac{AC}{XZ}$$

and Corresponding angles are equal

$\angle ABC = \angle XYZ$ (··)

$\angle BAC = \angle YXZ$ (·)

$\angle ACB = \angle XZY$ (···)

Example 1 Name the similar triangles. Write the letters so that equal angles appear in corresponding order.

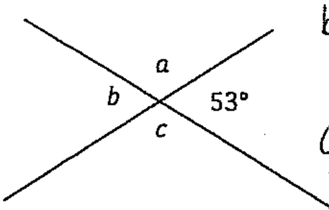
$\triangle ABC \sim \triangle QRP$

$\triangle PQR \sim \triangle CAB$

4.1

1. Find the measure of the unknown angle.

a)

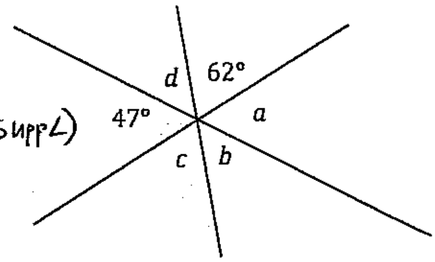


$b = 53^\circ$ (opp ∠)

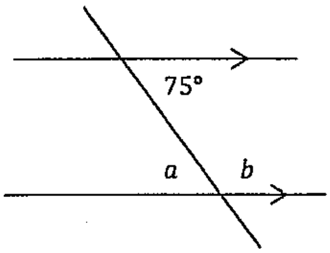
$a = 180 - 53 = 127^\circ$ (supp∠)

$c = 127^\circ$ (opp ∠)

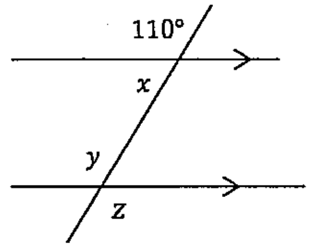
b)



b)



d)



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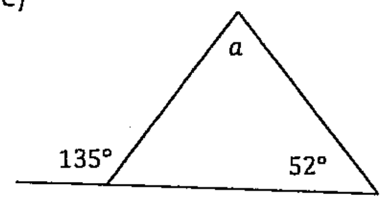
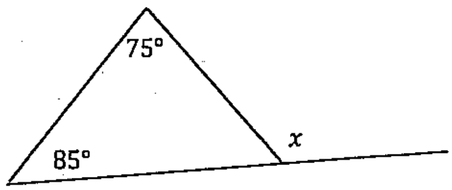
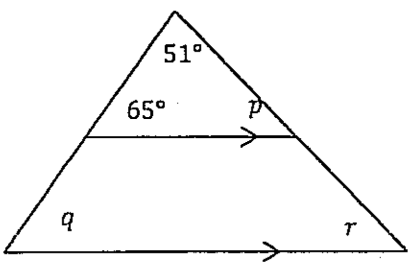


f)



g)



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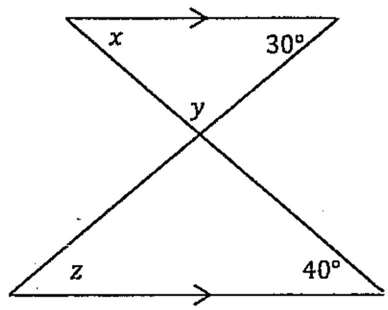
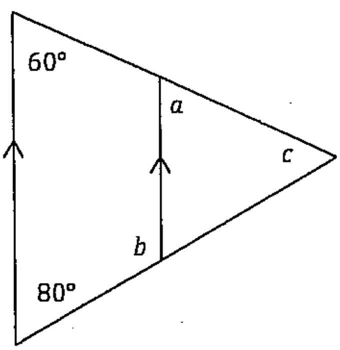
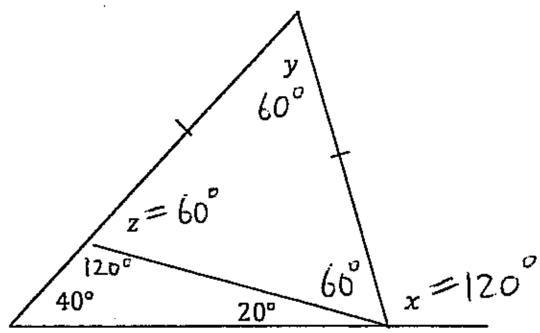


i)



j)

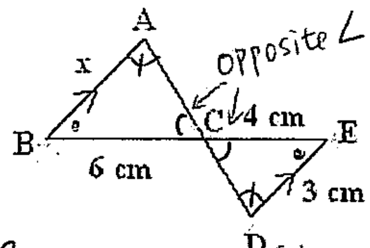


Answers:

1. a) a-127° (sup) b-53° (opp) c-127° (opp)
   b) a-47° (opp) b-71°(opp) c-62° (opp) d-71° (sup)
   c) a-75° (Z) b-105° (C)
   d) x–70° (sup) y–110° (F) z-110° (opp)
   e) a-83° (SATT)
   f) x-160° (sup)
   g) p-64° (SATT), q-65° (F), r-64° (F)
   h) x-40° (Z) y-110° (SATT) z-30° (Z)
   i) a-60° (F) b-100° (C) c-40° (SATT)
   j) x-100° (sup) y-60° (SATT) z-60° (sup)

Example 2 Determine the unknown measures

a)



$\triangle ABC \sim \triangle DEC$ (because of opposite∠ and alternate∠)

$$\frac{AB}{DE} = \frac{BC}{EC}$$

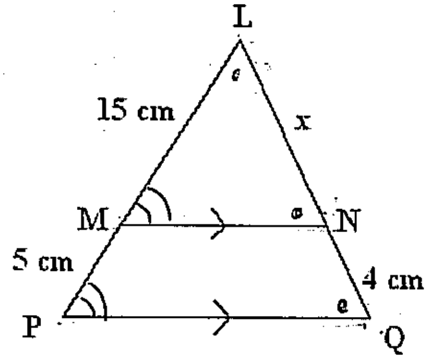
$$\frac{x}{3} = \frac{6}{4}$$

$$4x = 3 \times 6$$

$$\frac{4x}{4} = \frac{18}{4}$$

$x = \frac{9}{2}$ or 4.5 cm

b)



$\triangle LMN \sim \triangle LPQ$

$$\frac{LN \ (S\triangle)}{LQ \ (B\triangle)} = \frac{LM \ (S\triangle)}{LP \ (B\triangle)}$$

$$\frac{x}{(x+4)} = \frac{15}{20}$$

$$20x = 15(x+4)$$

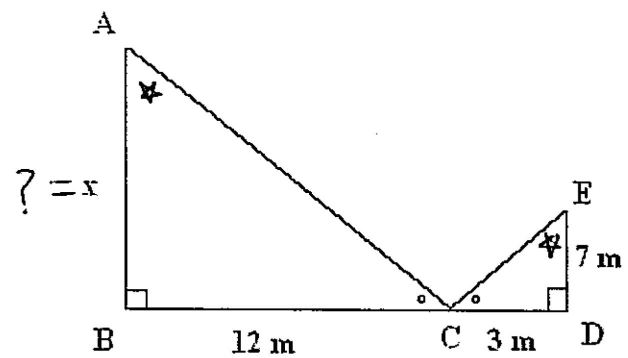
$$20x = 15x + 60$$

$$20x - 15x = 60$$

$$\frac{5x}{5} = \frac{60}{5}$$

$x = 12$ cm

Example 3 Find the value of each indicated quantity.



$\triangle ABC \sim \triangle EDC$

$$\frac{AB \ (B\triangle)}{ED \ (S\triangle)} = \frac{BC \ (\text{Big } \triangle)}{DC \ (\text{Small } \triangle)}$$

$$\frac{x}{7} = \frac{12}{3}$$

$$3x = 7 \times 12$$

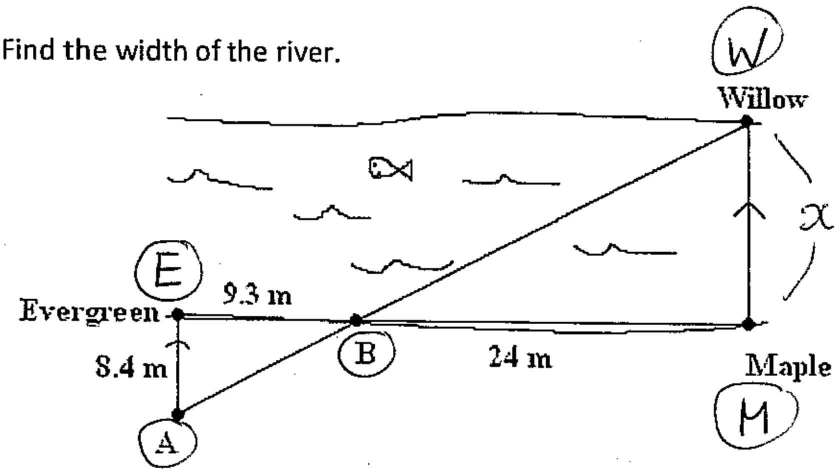
$$\frac{3x}{3} = \frac{84}{3}$$

$$x = 28$$

Example 4 Solve for an Unknown Side

To determine the width of a river, Naomi finds a willow tree and a maple tree that are directly across from each other on opposite shores. Using a third tree on the shoreline, Naomi plants two stakes, A and B, and measures the distances shown.

Find the width of the river.



$\triangle ABE \sim \triangle WBM$

$$\frac{WM}{AE} = \frac{BM}{BE}$$

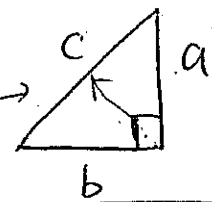
$$\frac{x}{8.4} = \frac{24}{9.3}$$

$$9.3x = 24 \times 8.4$$

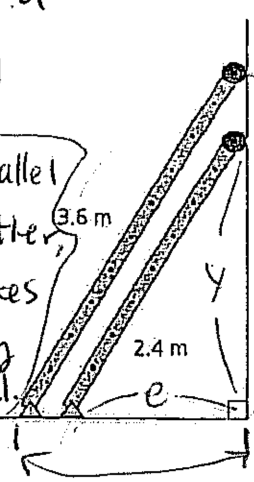
$$\frac{9.3x}{9.3} = \frac{201.6}{9.3}$$

$$x = 21.7\,m$$

$a^2+b^2=c^2$



7. A 3.6 m ladder is leaning against a wall with its base 2 m from the wall.
   (a) Use the Pythagorean theorem to determine how high up the wall the ladder reaches.
   (b) Suppose a 2.4 m ladder is placed against the wall parallel to the longer ladder. How far will it reach up the wall and how far will its base be from the wall?
   (c) **Communication:** Explain why the triangles formed by the ground, the wall, and the two ladders are similar. Because two ladders are parallel to each other, which makes corresponding ∠s to be equal.
   (d) **Knowledge and Understanding:** Suppose that an even longer ladder is placed parallel to the first two ladders, with its base 5 m from the wall. How long is this ladder?



$x = 2.99$

(d) $\frac{5}{2} = \frac{x}{3.6\,m}$

$2x = 5 \times 3.6$

$2x = 18$

$x = 9\,m$

(a) $x^2 + 2^2 = 3.6^2$

$x^2 = (3.6)^2 - 4$

$x^2 = 12.96 - 4 = 8.96$

$x = \sqrt{8.96} = 2.99\,m$

(b) $\frac{2.99}{y} = \frac{2}{e} = \frac{3.6}{2.4}$

$* \; 3.6e = 4.8$

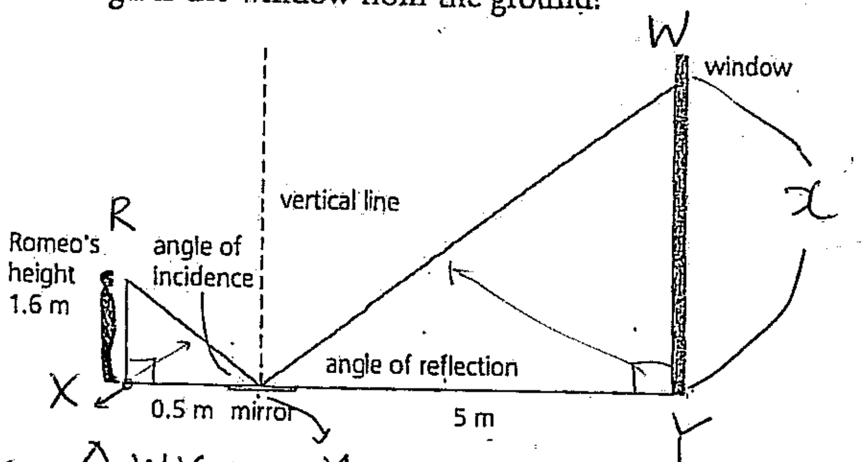
$\frac{3.6e}{3.6} = \frac{4.8}{3.6}$

(b) $e = 1.33\,m$

$* \; 3.6y = 2.99 \times 2.4$

$3.6y = 7.176 \rightarrow$ (b) $y = 2\,m$

13. Romeo uses a mirror to determine the height of Juliet's window. He knows that when light is reflected from a mirror, it makes the same angle on both sides of the point where it strikes the mirror. (The angle of incidence equals the angle of reflection.) How high is the window from the ground?



$\triangle RXM \sim \triangle WYM$

$$\frac{\text{Big}\triangle}{\text{Small}\triangle} \quad \frac{x}{1.6} = \frac{5}{0.5} \quad \frac{\text{Big}\triangle}{\text{S}\triangle}$$

$0.5x = 5 \times 1.6$

$0.5x = 8$

$x = 8 \div 0.5$

$x = 16\text{m}$

$\therefore$ The window is 16m high.

Homework: Worksheet

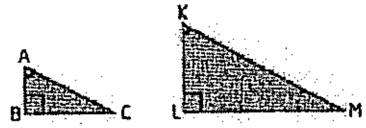
Similar Triangles WORKSHEET

McGraw-Hill Ryerson "Principles of Mathematics 10" p. 333-334 and 347-351

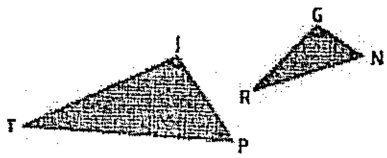
p. 333 – 334

5. Name the similar triangles in each case. Write the letters so that equal angles appear in corresponding order.

a)



b)



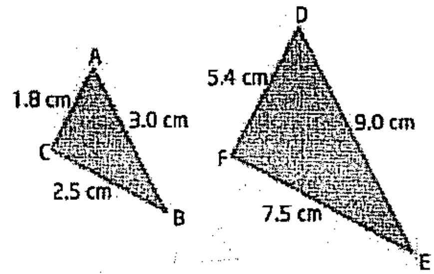
c)

d)

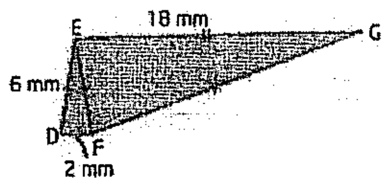
6. For each pair of similar triangles in question 5, write the equivalent ratios of side lengths.

8. Name a pair of similar triangles in each diagram and explain why they are similar.

a)



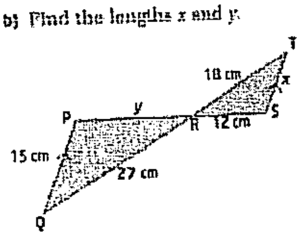
b)



p. 347 - 351

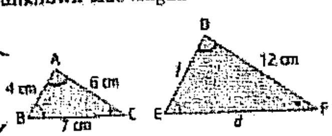
5. a) Show why $\triangle PQR$ is similar to $\triangle STR$.

b) Find the lengths x and y.

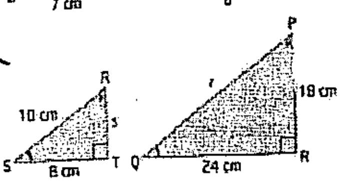


6. The triangles in each pair are similar. Find the unknown side lengths.

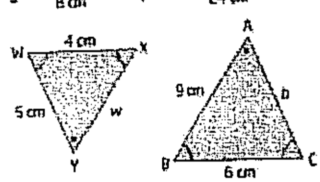
~~a)~~



~~b)~~

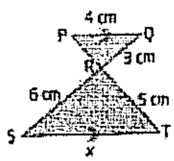


c)

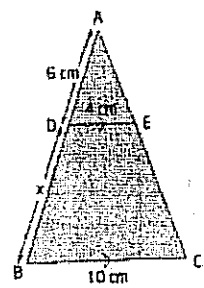


7. Find the length of x in each.

a)

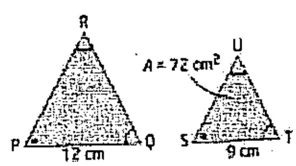


b)



For help with question 8, see Examples 2 and 3.

8. a) $\triangle PQR \sim \triangle STU$. Find the area of $\triangle PQR$.



b) $\triangle ABC \sim \triangle DEF$. Find the area of $\triangle ABC$.