

April 21

MCR3U
Ms. Kueh

Doubling, Half Life and Other fun problems!

* Unit Test will be on Monday, April 27

1. You see a great video on YouTube, and show it to 10 of your friends. After 1 hour each of your friends has shown the video to 10 of their friends who haven't seen it. After another hour, each of those people share it with another 10 people. The sharing of the video continues the same way. If a video is considered to be viral when one million people see it within one hour, how many hours does it take for this video to go viral?

<u>Time</u>	<u>Views</u>	
t=0	1 person	(0, 1)
t=1 hour	10 people	(1, 10)
t=2 hour	100 people	(2, 100)
t=3 hour	1000 people	(3, 1000)
t=4 hour	10,000 people	(4, 10,000)
t=5 hour	100,000 people	(5, 100,000)

Independent variable ?

time, so $x = \text{time}$

Dependent Variable ?

Number of viewers, so

$y = \# \text{ of viewers}$

* As x increase by 1, y increase by 10. $\rightarrow b = 10$

$$y = 10^x$$

* When $y = 1000,000 \rightarrow x = ? \rightarrow 1000,000 = 10^x$

$$10^6 = 10^x$$

$$x = 6$$

Create an equation for this situation.

\therefore It takes 6 hours for this video to go viral.

one on one

2. The Power of One! A school holds a Rock, Paper, Scissors Tournament to raise money for charity. 2048 students in the school participate.

a. After the first round of Rock, Paper, Scissors, how many students are left?

Only half of 2048 is winners or "survivors".

$2048 \div 2 = 1024$ survivors are left after first round.

b. After 8 rounds of rock paper scissors, how many students are left? Create an equation to help you solve this problem.

Round	Survivors	
0	2048	* Independent $V = \text{Round} = x \rightarrow \text{Let } x = \# \text{ of rounds}$
1	1024	* Dependent $V = \# \text{ of survivors} = y \rightarrow \text{Let } y = \# \text{ of survivors}$
2	512	* As x increases by 1, y decrease by $\frac{1}{2} \rightarrow b = \frac{1}{2}$
3	256	* When $x=0, y=2048 \rightarrow a=2048$
4	128	$\therefore y = 2048 \left(\frac{1}{2}\right)^x$

c. How many rounds are played before the winner of the school is determined?

b. when $x=8, y=?$
 $y = 2048 \left(\frac{1}{2}\right)^8$
 $y = 8$
 $\therefore 8$ students are left after 8 rounds.

$y=1 \rightarrow$ sub into equation and solve $x=?$
 $1 = 2048 \left(\frac{1}{2}\right)^x$
 $\frac{1}{2048} = \left(\frac{1}{2}\right)^x$
 $\frac{1}{2^{11}} = \left(\frac{1}{2}\right)^x$
 $\left(\frac{1}{2}\right)^{11} = \left(\frac{1}{2}\right)^x \therefore x=11$
 $\therefore 11$ rounds are played before the winner is determined.

3. A silly joke has started about Ms. Kueh loving e-"kueh"-tions. One student tells 2 friends. Each of those friends tells 2 friends. If it takes 30 minutes to find 2 friends and tell them, how long before 512 people are told the joke within a span of 30 minutes?

Time	People	
0	1	* Independent $V = \text{Time} =$ Let x be # of time in minutes
30 min	2	* Dependent $V = \# \text{ of people} =$ Let y be # of people who told jokes.
60 min	4	Equation: $y = 2^{\frac{x}{30}}$ $x=?$ when $y = 512$
90 min	8	

Your x increase by 30 minutes

$512 = 2^{\frac{x}{30}}$
 $2^9 = 2^{\frac{x}{30}}$
 $30 \times 9 = \frac{x}{30} \times 30$
 $270 = x$

\therefore After 270 minutes (or 4.5 hours) 512 people are told about this joke.

4. A population of 10 bacteria divide every 2 hours. Assuming this trend continues and that no bacteria die, what will the population be 8 hours later?

Time # of bacteria

0 hour

10 $\rightarrow a=10$

2

20

4

40

6

80

* Independent $v = \text{time} \Rightarrow$ Let x be # of hours

* Dependent $v = \# \text{ of bacteria} \Rightarrow$ Let y be # of

\uparrow by 2 = b bacteria's population

$$y = 10(2)^{\frac{x}{2}} \quad \text{when } x=8, y=?$$

$$y = 10(2)^{\frac{8}{2}} \rightarrow y = 10(2)^4 = 160$$

\therefore After 8 hours, the pop of bacteria will be 160.

x increase by 2 hours $\rightarrow \frac{x}{2}$

5. A 200 g sample of polonium-210 has a half-life of 138 days. This means that every 138 days, the amount of polonium left in a sample is half of the original amount.

Time Amount (Po)

0 days

200g = a

Let x be the time in days

138 days

100g

$\downarrow \frac{1}{2} = b$

Let y // amount of polonium

276 days

50g

- b. Use the equation to determine the mass of polonium-210 left after 5 years.

$$* 5 \text{ years} = 5 \times 365 = 1825 \text{ days}$$

$$\text{when } x = 1825 \text{ days, } y = ?$$

$$y = 200 \cdot \left(\frac{1}{2}\right)^{\frac{1825}{138}}$$

$$y = 200 \cdot \left(\frac{1}{2}\right)^{13.224638}$$

$$y = 0.020894$$

\therefore After 5 years, the polonium-210's population is 0.0209

x increase by 138 days

$$\text{so exponent} = \frac{x}{138}$$

$$y = 200 \cdot \left(\frac{1}{2}\right)^{\frac{x}{138}}$$

6. A research assistant made 160 mg of radioactive sodium and found that there was only 20 mg left 45 h later. What is the half-life of radioactive sodium?

Given by the question: $y = 160 \left(\frac{1}{2}\right)^{\frac{x}{h}}$ \rightarrow implies that $b = \frac{1}{2}$ $\rightarrow h$ is only unknown.

When $x = 45 \text{ h}$, $y = 20 \text{ mg}$ so we will sub (45, 20) into equation:

$$20 = 160 \left(\frac{1}{2}\right)^{\frac{45}{h}}$$

$$\underline{20 = 160 \left(\frac{1}{2}\right)^{\frac{45}{h}}}$$

$$\frac{1}{8} = \left(\frac{1}{2}\right)^{\frac{45}{h}}$$

$$\left(\frac{1}{2}\right)^3 = \left(\frac{1}{2}\right)^{\frac{45}{h}}$$

$$3 = \frac{45}{h}$$

$$3h = 45$$

$$h = 15$$

\therefore The half life is 15 hours.

Appreciation and Depreciation

Recall: Calculate the total cost, with tax, of a book that is priced at \$29.99.

Was your calculation one step? If not, figure out how to calculate the total cost in one step.

Example 1 Compound Interest

In 2000, \$1000 was invested at a rate of 6% per year for 4 years.

a) How much money was there after 4 years?

Number of years	Money
0	
1	
2	
3	
4	

b) Write an equation to model this growth.