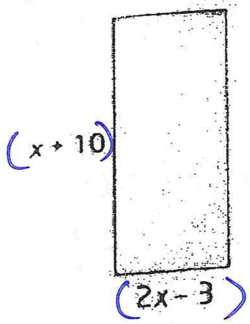
May 4 (#8, 9, 10, 15)

8. A rectangle has dimensions $x + 10$ and $2x - 3$. Determine the value of x that gives an area of 54 cm^2.

Area = $l \times w$



9. Write a quadratic equation in factored form for each situation.

a) The roots of the equation are 5 and 4.

b) The roots of the equation are -2 and 3.

10. a) Write a quadratic equation in the form $ax^2 + bx + c = 0$ with roots of 6 and -7.

b) What would happen to the roots if you multiplied both sides of the equation in part a) by 3? Explain.

15. For the equation $3n^2 = 15n$, Chris suggested dividing both sides by $3n$, leaving $n = 5$. Are there any other values that satisfy this equation? What is wrong with Chris's method? What is a more appropriate method?

$$0 = \left(x - \frac{2}{3}\right)\left(x + \frac{4}{5}\right)$$

$$0 = x^2 + \frac{4}{5}x - \frac{2}{3}x - \frac{8}{15}$$

$$0 = x^2 + \frac{12x}{15} - \frac{10x}{15} - \frac{8}{15}$$

$$0 = x^2 + \frac{2x}{15} - \frac{8}{15}$$

×15 ×15 ×15 ×15

$$0 = 15x^2 + 2x - 8$$

May 5

11. Write a quadratic equation with roots of $\frac{2}{3}$ and $-\frac{4}{5}$ in the form $ax^2 + bx + c = 0$, where a, b, and c are integers.

Answers

8. 3.5 cm

9. a) $(x - 5)(x - 4) = 0$ b) $(x + 2)(x - 3) = 0$

10. a) $x^2 + x - 42 = 0$

b) the roots remain the same because the quadratic equation is equivalent to $x^2 + x - 42 = 0$ and will have the same factors.

15. $n = 0$ will also satisfy the equation. If Chris wants to divide out a common factor, it should not contain any variables. Chris should subtract $15n$ from both sides of the equation by 3 and then solve by factoring.

11. $15x^2 + 2x - 8 = 0$

8. $54 = l \times w$

$54 = (x + 10)(2x - 3)$

$54 = 2x^2 - 3x + 20x - 30$

$0 = 2x^2 - 3x + 20x - 30 - 54$

$0 = 2x^2 + 17x - 84$

$0 = 2x^2 + 24x - 7x - 84$

$0 = 2x(x + 12) - 7(x + 12)$

$0 = (x + 12)(2x - 7)$

$ac = 2 \times -84$
$= -168$
$b = 17$
$24 \times -7 = -168$
$24 - 7 = 17$

$x + 12 = 0$ or $2x - 7 = 0$

$x = -12$ $\quad 2x = 7$

$x = \frac{7}{2}$

$\therefore x = \frac{7}{2}$ is the only solution (because $x = -$ leads to negative length)

May 5

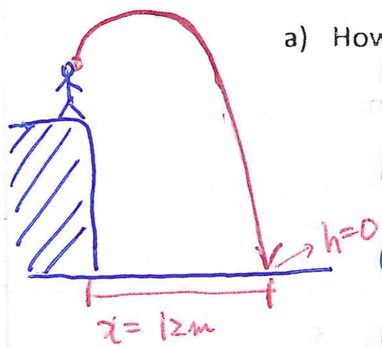
MPM2D

More Applications – Factor First!

Example 1

canyon or valley

The path of a stone thrown into a ravine is modelled by the quadratic relation $h(x) = -x^2 + 5x + 84$, where x represents the distance, in metres, travelled horizontally and h represents the height, in metres, above the surface of the river at the bottom of the ravine.



a) How far does the stone travel horizontally before it hits the water?

Let x represent distance (m) ↳ when $h=0$

Let h represent heigh (m) $x=?$

$0 = -x^2 + 5x + 84$ (we substitute $h=0$ into the equation)

$0 = -(x^2 - 5x - 84)$

$ac = 1 \times -84 = -84$

$b = -5$

$-12 \times 7 = -84$

$-12 + 7 = -5$

$\cancel{0 = -(x^2 - 12x + 7x - 84)}$

$\cancel{0 = -[x(x-12) + 7(x-12)]}$

$0 = -(x-12)(x+7)$

$x - 12 = 0$ or $x + 7 = 0$

$x = 12$ $\quad x = -7$ → reject it because distance can not be negative.

∴ The stone travels 12 m before it hits the water.

b) How far has the stone travelled horizontally when it reaches maximum height?

Find x coordinate of the vertex

= Add two x intercepts and then divide it by 2.

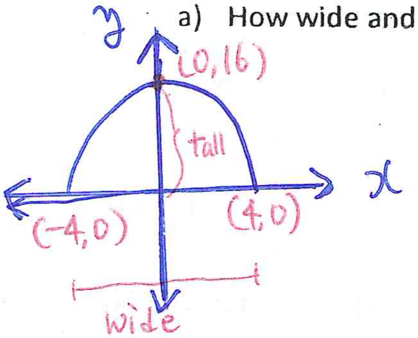
* x coordinate of vertex $= \dfrac{12 + (-7)}{2} = \dfrac{5}{2} = 2.5\text{m}$

∴ The stone travelled 2.5 m when it reaches maximum height.

Example 2 Design an Arch

To commemorate the 100th anniversary of the Newtonville Fair, an entrance arch will be built. The design engineer uses the equation $h(d) = -d^2 + 16$ to model the arch, where h is the height, in metres, above the ground and d is the horizontal distance, in metres, from the centre of the arch.

a) How wide and how tall is the arch?



h = height , d = horizontal distance

$h(d) = -d^2 + 16$

$0 = -d^2 + 16$

$0 = -(d^2 - 16)$

$0 = -(d^2 - 4^2) \rightarrow a = d, \; b = 4$

$a^2 - b^2 = (a+b)(a-b)$

$0 = -(d+4)(d-4)$

$d + 4 = 0 \qquad d - 4 = 0$

$d = -4 \qquad d = 4$

∴ The width of the arch is 8 m

* x coord of vertex

$\frac{-4+4}{2} = \frac{0}{2} = 0$

× y coord of vertex

$h(d) = -0^2 + 16$ → $h = 0 + 16 = 16$

∴ The height of the arch is 16m.

b) For what values of d is the relation valid? Explain.

Domain = ? What are possible x values?

$D = \{d \in \mathbb{R}, -4 \leq d \leq 4\}$

Example 3

The sum of the squares of two consecutive even numbers is 164. Find the integers.

2, 4, 6, 8, 10

yes   yes

Let First even # be x

Let second even # be $x+2$

$$x^2 + (x+2)^2 = 164$$

$$x^2 + x^2 + 2x + 2x + 4 = 164$$

$$2x^2 + 4x + 4 - 164 = 0$$

$$2x^2 + 4x - 160 = 0$$

$$2(x^2 + 2x - 80) = 0$$

$$2(x+10)(x-8) = 0$$

* $(x+2)^2 = (x+2)(x+2)$

$= x^2 + 2x + 2x + 4$

$ac = -80$

$b = 2$

$10 \times -8 = -80$

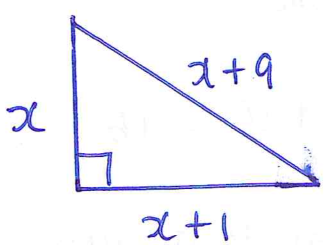
$10 - 8 = 2$

$x+10=0$	$x-8=0$
$x=-10$	$x=8$
$-10+2=-8$	$8+2=10$

∴ The two integers are either −8, −10 or 8, 10

Example 4

One leg of a right triangle is 1 *cm* longer than the other leg. The length of the hypotenuse is 9 *cm* greater than that of the shorter leg. Find the length of the three sides.



$a^2 + b^2 = c^2$ Hypotenuse (opposite from right∠)

$$x^2 + (x+1)^2 = (x+9)^2$$

$$x^2 + [(x+1)(x+1)] = (x+9)(x+9)$$

$$x^2 + [x^2 + x + x + 1] = x^2 + 9x + 9x + 81$$

$$x^2 + x^2 + 2x + 1 - x^2 - 18x - 81 = 0$$

$$x^2 - 16x - 80 = 0$$

$$(x+4)(x-20) = 0$$

$= -80$ $-20 \times 4 = -80$

$b = -16$ $-20 + 4 = -1$

$x+4=0$	$x-20=0$
$x=-4$	$x=20$
	$x+1=21$
	$x+9=29$

We reject −4 because you can not have negative side.

∴ The 3 sides are 20, 21cm and 29.

Homework: Worksheet (pg. 280) # (7), 14, 17   (pg. 290) #8, 9, 10, 12, (16)
(pg. 312) #5, 10   Brackets are optional extra practice

TIPS practice: (pg. 280) #16, 21b   (pg. 291) #19, 20, 21