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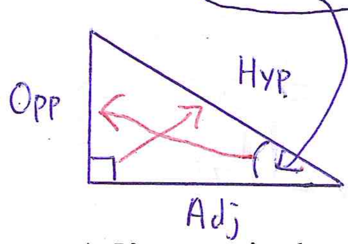
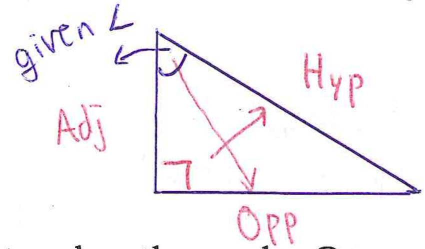
Primary and Reciprocal Ratios

Right Angle Triangles

From a given angle, we can name the sides of a right angle triangle:





Adjacent is the side that touches the angle. **Opposite** is the side that does not touch the angle. (opposite from the given angle)

***Note:** The opposite and adjacent sides depend on the angle! If we look at the same triangle but a different angle, the opposite and adjacent sides will be different, but the Hypotenuse always stays the same!

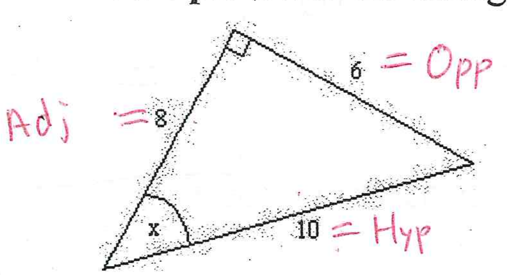
Primary Trigonometric Ratios

In a **right triangle**, the angles and the lengths of sides are related by certain ratios:

Sine $= \dfrac{\text{Opp}}{\text{Hyp}}$ **Cosine** $= \dfrac{\text{Adjacent}}{\text{Hyp}}$ **Tangent** $= \dfrac{\text{Opp}}{\text{Adj}}$

$$\boxed{S\frac{O}{H} \quad C\frac{A}{H} \quad T\frac{O}{A}}$$

Example 1 For the triangle below, find x



a) $\sin x = \frac{6}{10}$ $(S\frac{O}{H})$

$x = \sin^{-1}(0.6)$

$x = 37°$

b) $\cos x = \frac{8}{10}$ $(C\frac{A}{H})$

$x = \cos^{-1}\left(\frac{8}{10}\right) \approx 37°$

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Primary and Reciprocal Ratios

$T \frac{O}{A}$

a) sin x

b) cos x

c) tan x $= \frac{6}{8}$

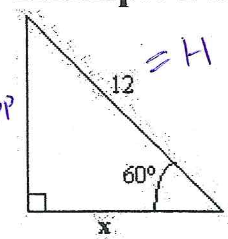
$\frac{\quad}{\text{Tan}} \qquad \frac{\quad}{\text{Tan}}$

$x = \text{Tan}^{-1}\left(\frac{6}{8}\right)$

$x = 37°$

Example 2 Find the side length

Opp



$= H$

$= \text{Adj}$

$\text{cah} = \cos 60° = \frac{a}{H}$

$12 \times \cos 60° = \frac{x}{12} \times 12$

$6 = x$

$\therefore x = 6$

③ $\text{Tan}\ \theta = \frac{\text{Opp}}{\text{Adj}}$

Primary Trig ratios ① $\sin\theta = \frac{\text{Opp}}{\text{Hyp}}$ ② $\cos\theta = \frac{\text{Adj}}{\text{Hyp}}$

Reciprocal Trig Ratios

$\text{cosecant}\ \theta = \frac{1}{\sin\theta} = \frac{1}{\frac{\text{Opp}}{\text{Hyp}}} = \frac{\text{Hyp}}{\text{Opp}}$

$\text{Secant}\ \theta = \frac{1}{\cos\theta} = \frac{\text{Hyp}}{\text{Adj}}$

$\boxed{\text{cotangent}\ \theta = \frac{\text{Adj}}{\text{Opp}} = \frac{1}{\text{Tan}\ \theta}}$

* Calculators don't have reciprocal trig buttons, so if you to evaluate

$= \quad \sec 20° = \frac{1}{\cos 20°} = 1.06$

$\text{cosecant}\ 20° = \frac{1}{\sin 20°} = 2.92$

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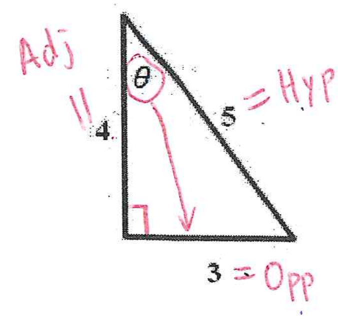
Primary and Reciprocal Ratios

Example 3

a) Determine csc , sec and cot



$\csc\theta = \dfrac{\text{Hyp}}{\text{Opp}}$ (Opposite of SOH) $= \dfrac{5}{3}$

$\sec\theta$ (Opposite of CAH) $= \dfrac{5}{4} = \dfrac{H}{A}$

$\cot\theta$ (Opp of Tangent) $= \dfrac{A}{O} = \dfrac{4}{3}$

b) Calculate θ to the nearest degrees.

$\csc\theta = \dfrac{5}{3}$ * $\csc\theta = \dfrac{1}{\sin\theta}$

$\sin\theta$ is opposite of $\csc\theta$

$\sin\theta = \dfrac{3}{5}$

$\theta = \sin^{-1}\left(\dfrac{3}{5}\right) = 36.9^\circ$

"**Solve**" a triangle means find the missing angles and sides.

<u>Use your math tools:</u>

1) Trig ratios — Use both primary and reciprocal ratios.

2) Pythagorean Theorem $a^2 + b^2 = c^2$

3) Sum of 3 Angles in a Triangle is 180°

~~Home~~ Class work: #1, 3-6, 8, ~~10, 12~~, ~~14~~ (Worksheet)

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Primary and Reciprocal Ratios

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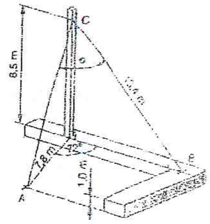


UNIT ~~4~~ 5 – TRIGONOMETRIC RATIOS

Expectations

By the end of this unit, you will be able to:

1. write the primary and reciprocal trig ratios in terms of the sides of a right triangle.

2. determine the exact values of the sine, cosine, and tangent of the special angles: 0°, 30°, 45°, 60°, and 90°.

3. determine the values of the sine, cosine, and tangent of angles related to special angles from 0° to 360°.

4. sketch the graphs of $f(x) = \sin x$, $f(x) = \cos x$, and $f(x) = \tan x$.

5. determine the measure of two angles from 0° to 360° that have the same trigonometric ratio.

6 solve simple trig equations using a graph or the CAST Rule.

7. solve problems involving triangles in two-dimensional settings using the primary trig ratios, the cosine law, and the sine law.

8. solve problems in three-dimensional settings using the primary trig ratios, the cosine law, and the sine law.

Homework

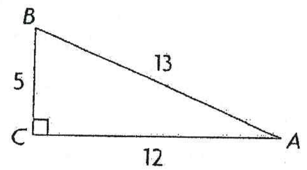
The schedule and homework assignments below are subject to change at the teacher's discretion.

Day	Topic	Homework
1	Primary and Reciprocal Trig Ratios	Worksheet #1, 3-6, 8, 10-12, 14
2	Special Angles	Worksheet #4-12
3	Angles between 0° and 90° in the - Plane	Pg 237 C3, C4, #1ad, 2e, 3af, 5, 20*, 21, 22
4	Angles between 90° and 360°	Pg. 238 #1bce, 2abc, 3bcde Worksheet
5	Graphs of Sine, Cosine, and Trig Functions	Worksheet
6 **Quiz Day 2-4**	Solving Trig Equations	Worksheet
7	The Sine and Cosine Laws Ambiguous Case	Worksheet
8	Solving 2-D Problems	Pg. 254 #3-5,7, 10, 11, 14, 18, 21 Read textbook examples 1, 2 and 3 on page 250, if necessary.
9 **Quiz Day 5-7**	Solving 3-D Problems Using Trigonometry	Pg. 265 #1-3, 5-7 Thinking Pg. 268 14, 15, 17, 19 Read textbook examples 1, 2 and 3 on page 262, if necessary.
10	Review	Pg. 276 #1-12 Pg. 278 #1-10
11	Test	

Homew : April 28

CHECK Your Understanding

1. Given $\triangle ABC$, state the six trigonometric ratios for $\angle A$.



2. State the reciprocal trigonometric ratios that correspond to $\sin\theta = \frac{8}{17}$, $\cos\theta = \frac{15}{17}$, and $\tan\theta = \frac{8}{15}$.

3. For each primary trigonometric ratio, determine the corresponding reciprocal ratio.

a) $\sin\theta = \frac{1}{2}$

b) $\cos\theta = \frac{3}{4}$

c) $\tan\theta = \frac{3}{2}$ $\rightarrow \cot\theta = \frac{2}{3}$

d) $\tan\theta = \frac{1}{4}$

4. Evaluate to the nearest hundredth.

a) $\cos 34°$ b) $\sec 10°$ c) $\cot 75°$ d) $\csc 45°$

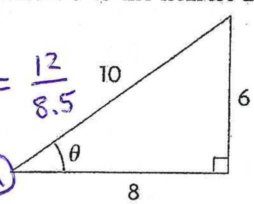
0.83 $= \frac{1}{\cos 10°} = 1.02$ $= \frac{1}{\tan 75°} = 0.268$ $= \frac{1}{\sin 45°} = 1.41$

PRACTISING

5. a) For each triangle, calculate $\csc\theta$, $\sec\theta$, and $\cot\theta$.

K b) For each triangle, use one of the reciprocal ratios from part (a) to determine θ to the nearest degree.

i)



$\csc\theta = \frac{H}{O} = \frac{12}{8.5}$

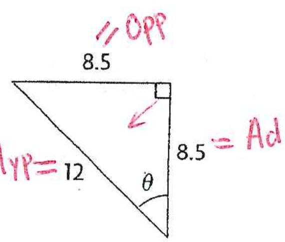
$\csc\theta = \frac{12}{8.5}$ (a)

$\theta = \frac{8.5}{12}$ (ii)

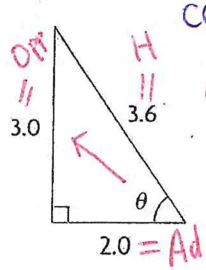
$= \sin^{-1}\frac{8.5}{12}$

$= 45.1°$ (b)

ii)



iii)



$\cot\theta = \frac{A}{O} = \frac{2}{3}$

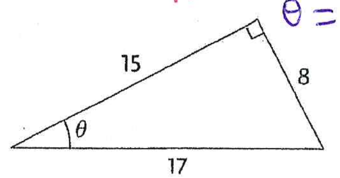
$\cot\theta = \frac{2}{3}$ (A) iii)

$\tan\theta = \frac{3}{2}$

$\theta = \tan^{-1}\left(\frac{3}{2}\right)$

$\theta = 56°$ (B) iii)

iv)



6c) $\sec\theta = 1.4526$

$\cos\theta = \frac{1}{1.4526}$

cos cos

$\theta = \cos^{-1}\left(\frac{1}{1.4526}\right)$ $\rightarrow \theta = 46°$

6. Determine the value of θ to the nearest degree.

a) $\cot\theta = 3.2404$

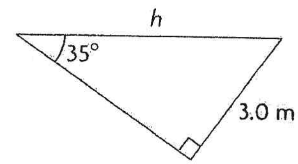
b) $\csc\theta = 1.2711$

c) $\sec\theta = 1.4526$

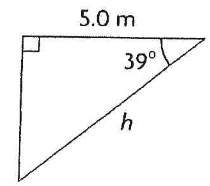
d) $\cot\theta = 0.5814$

7. For each triangle, determine the length of the hypotenuse to the nearest tenth of a metre.

a)

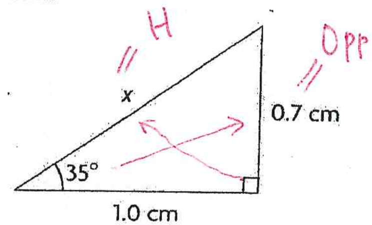


b)

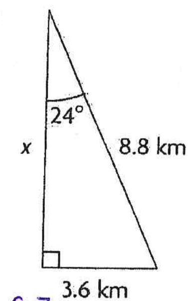


8. For each triangle, use two different methods to determine x to the nearest tenth of a unit.

a)



b)



a) $\sin 35° = \frac{O}{H} = \frac{0.7}{x}$

$x = \frac{0.7}{\sin 35°} = 1.22\text{ cm}$

9. Given any right triangle with an acute angle θ,

a) explain why $\csc\theta$ is always greater than or equal to 1

b) explain why $\cos\theta$ is always less than or equal to 1

10. Given a right triangle with an acute angle θ, if $\tan\theta = \cot\theta$, describe what this triangle would look like.

11. A kite is flying 8.6 m above the ground at an angle of elevation of 41°.

A Calculate the length of string, to the nearest tenth of a metre, needed to fly the kite using

a) a primary trigonometric ratio

b) a reciprocal trigonometric ratio

12. A wheelchair ramp near the door of a building has an incline of 15° and a run of 7.11 m from the door. Calculate the length of the ramp to the nearest hundredth of a metre.

13. The hypotenuse, c, of right $\triangle ABC$ is 7.0 cm long. A trigonometric ratio for angle A is given for four different triangles. Which of these triangles has the greatest area? Justify your decision.

a) $\sec A = 1.7105$ c) $\csc A = 2.2703$
b) $\cos A = 0.7512$ d) $\sin A = 0.1515$

14. The two guy wires supporting an 8.5 m TV antenna each form an angle of 55° with the ground. The wires are attached to the antenna 3.71 m above ground. Using a reciprocal trigonometric ratio, calculate the length of each wire to the nearest tenth of a metre. What assumption did you make?

15. From a position some distance away from the base of a flagpole, Julie estimates that the pole is 5.35 m tall at an angle of elevation of 25°. If Julie is 1.55 m tall, use a reciprocal trigonometric ratio to calculate how far she is from the base of the flagpole, to the nearest hundredth of a metre.

16. The maximum grade (slope) allowed for highways in Ontario is 12%.
a) Predict the angle θ, to the nearest degree, associated with this slope.
b) Calculate the value of θ to the nearest degree.
c) Determine the six trigonometric ratios for angle θ.

17. Organize these terms in a word web, including explanations where appropriate.

sine	cosine	tangent	opposite
cotangent	hypotenuse	cosecant	adjacent
secant	angle of depression	angle	angle of elevation

Extending

18. In right $\triangle PQR$, the hypotenuse, r, is 117 cm and $\tan P = 0.51$. Calculate side lengths p and q to the nearest centimetre and all three interior angles to the nearest degree.

19. Describe the appearance of a triangle that has a secant ratio that is greater than any other trigonometric ratio.

20. The tangent ratio is undefined for angles whose adjacent side is equal to zero. List all the angles between 0° and 90° (if any) for which cosecant, secant, and cotangent are undefined.

1. $\sin A = \frac{5}{13}$, $\cos A = \frac{12}{13}$, $\tan A = \frac{5}{12}$,
$\csc A = \frac{13}{5}$, $\sec A = \frac{13}{12}$, $\cot A = \frac{12}{5}$

2. $\csc \theta = \frac{17}{[illegible]}$, $\sec \theta = \frac{17}{15}$, $\cot \theta = \frac{15}{8}$

3. a) $\csc \theta = 2$ b) $\sec \theta = \frac{4}{3}$ c) $\cot \theta = \frac{2}{3}$ d) $\cot \theta = 4$

4. a) 0.83 b) 1.02 c) 0.27 d) 1.41

5. a) i) $\csc \theta = \frac{5}{3}$, $\sec \theta = \frac{5}{4}$, $\cot \theta = \frac{4}{3}$

ii) $\csc \theta = \frac{12}{8.5}$, $\sec \theta = \frac{12}{8.5}$, $\cot \theta = 1$

iii) $\csc \theta = \frac{3.6}{3}$, $\sec \theta = \frac{3.6}{2}$, $\cot \theta = \frac{2}{3}$

iv) $\csc \theta = \frac{17}{8}$, $\sec \theta = \frac{17}{15}$, $\cot \theta = \frac{15}{8}$

b) i) 37° ii) ~~56°~~ 45° iii) ~~45°~~ 56° iv) 28°

6. a) 17° b) 52° c) 46° d) 60°

7. a) 5.2 m b) 6.4 m

8. a) 1.2 cm b) 8.0 km

9. a) For any right triangle with acute angle θ, $\csc \theta = \frac{\text{hypotenuse}}{\text{opposite}}$.
Case 1: If neither the adjacent side nor the opposite side is zero, the hypotenuse is always greater than either side and $\csc \theta > 1$.
Case 2: If the adjacent side is reduced to zero, each time you calculate $\csc \theta$, you get a smaller and smaller value until $\csc \theta = 1$.
Case 3: If the opposite side is reduced to zero, each time you calculate $\csc \theta$, you get a greater and greater value until you reach infinity.
So for all possible cases in a right triangle, cosecant is always greater than or equal to 1.

b) For any right triangle with acute angle θ, $\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$.
Case 1: If neither the adjacent side nor the opposite side is zero, the hypotenuse is always greater than either side and $\cos \theta < 1$.
Case 2: If the opposite side is reduced to zero, each time you calculate $\cos \theta$, you get a greater and greater value until $\cos \theta = 1$.
Case 3: If the adjacent side is reduced to zero, each time you calculate $\cos \theta$, you get a smaller and smaller value until $\cos \theta = 0$.
So for all possible cases in a right triangle, cosine is always less than or equal to 1.

10. $\theta = 45°$ and adjacent side = opposite side

11. a) and b) 13.1 m

12. 7.36 m

13. (b) a right triangle with two 45° angles would have the greatest area, at an angle of 41°, (b) is closest to 45° and will therefore have the greatest area of those triangles.

14. 4.5 m

15. 8.15 m

16. a) Answers will vary. For example, 10° b) 7°
c) $\sin \theta = \frac{3}{\sqrt{634}}$, $\cos \theta = \frac{25}{\sqrt{634}}$, $\tan \theta = \frac{3}{25}$,
$\csc \theta = \frac{\sqrt{634}}{3}$, $\sec \theta = \frac{\sqrt{634}}{25}$, $\cot \theta = \frac{25}{3}$

18. $p = 53$ cm, $q = 104$ cm, $\angle P = 27°$, $\angle Q = 63°$

19. Since $\sec \theta = \frac{\text{hypotenuse}}{\text{adjacent}}$, the adjacent side must be the smallest side.

20. (csc and cot) 0°, (sec) 90°