

April 29

MCR3U

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### Special Triangles

Sometimes it is necessary to know exact trig ratios.

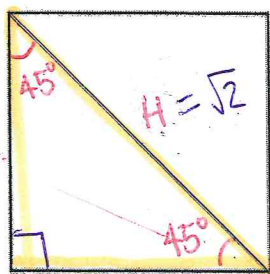
There are 2 special triangles that you can use to find exact angles and sides:

$$A : O : H$$

$$1 : 1 : \sqrt{2}$$

45° Right Triangle's ratio of 3 sides

- start with a square
- divide in half on the diagonal



Use Pythagorean Theorem to find the length of the diagonal

$$1^2 + 1^2 = H^2$$

$$1 + 1 = H^2$$

$$\sqrt{2} = \sqrt{H^2}$$

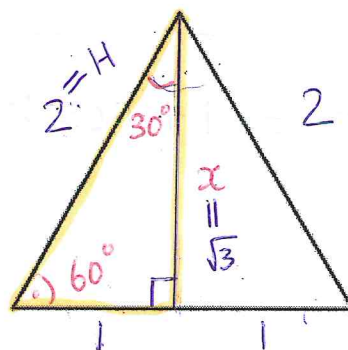
$$\therefore H = \sqrt{2}$$

$$A : O : H$$

$$1 : \sqrt{3} : 2$$

30°/60° Right Triangle's ratio of 3 sides

- start with an equilateral triangle = all  $\angle$ s = 60°
- divide in half



Use Pythagorean Theorem to find the length of the vertical

$$2^2 = x^2 + 1^2$$

$$4 = x^2 + 1$$

$$\sqrt{3} = \sqrt{x^2}$$

$$\sqrt{3} = x$$

Example 1 Find the exact value of

S O H

$$a) \sin 45^\circ = \frac{1 \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$b) \tan 60^\circ = \frac{\sqrt{3}}{1} = \sqrt{3}$$

TOA  
↑

$$c) \cos 30^\circ = \frac{\sqrt{3}}{2}$$

CAH  
↑

Example 2 Determine the exact value of

~~$$(\sin 45^\circ)(\cos 45^\circ) + (\sin 30^\circ)(\sin 60^\circ)$$~~

$$= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}\right) + \left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$$

$$= \frac{1}{2} + \frac{\sqrt{3}}{4}$$

$$= \frac{2}{4} + \frac{\sqrt{3}}{4} = \frac{2 + \sqrt{3}}{4}$$

HW: Worksheet # 4-12

$\theta$	$\sin \theta$	$\cos \theta$	$\tan \theta$
$30^\circ$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
$45^\circ$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
$60^\circ$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$

Example 3 Show (or prove) that

$$\frac{(\sin^2 30^\circ)}{1 - \cos 30^\circ} = 1 + \cos 30^\circ$$

\*For every proof, you need to keep LS and RS separate! Write a therefore statement at the end.

$$= (\sin 30^\circ)^2$$

LS

RS

$$\frac{(\sin^2 30^\circ)}{1 - \cos 30^\circ}$$

$$= \frac{\left(\frac{1}{2}\right)^2}{1 - \frac{\sqrt{3}}{2}}$$

$$= \frac{\frac{1^2}{2^2}}{\frac{2}{2} - \frac{\sqrt{3}}{2}}$$

$$= \frac{\frac{1}{4}}{\left(\frac{2-\sqrt{3}}{2}\right)} = \frac{1}{4} \div \left(\frac{2-\sqrt{3}}{2}\right)$$

$$= \frac{1}{4} \times \left(\frac{2}{2-\sqrt{3}}\right) = \frac{1(2+\sqrt{3})}{2(2-\sqrt{3})(2+\sqrt{3})}$$

$$= \frac{2+\sqrt{3}}{2(4+2\sqrt{3}-2\sqrt{3}-3)}$$

$$= \frac{2+\sqrt{3}}{2} = 1 + \frac{\sqrt{3}}{2}$$

$$1 + \cos 30^\circ$$

$$= 1 + \frac{\sqrt{3}}{2}$$

$$= \frac{2}{2} + \frac{\sqrt{3}}{2}$$

$$= \frac{2+\sqrt{3}}{2}$$

$$\therefore \text{LS} = \text{RS}$$

$$(a+b)(a-b) = a^2 - b^2$$

$$(a-b)(a-b) = a^2 + b^2 - 2ab$$

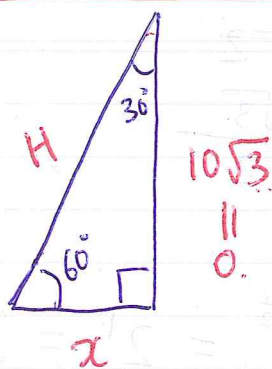
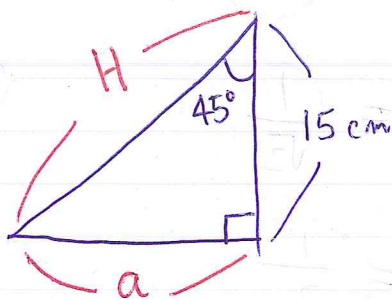
$$a : O : H$$

$$1 : 1 : \sqrt{2} \rightarrow \text{small } \Delta \text{ ratio}$$

$$15 : 15 : \bigcirc \rightarrow \text{Big } \Delta \text{ ratio}$$

$$H = \sqrt{2} \times 15 = 15\sqrt{2}$$

$$\therefore H = 15\sqrt{2} \text{ and } a = 15$$



$$1 : \sqrt{3} : 2$$

Small  $\Delta$

$$A \quad O \quad H$$

$$x : 10\sqrt{3} : H$$

$$10 = 1 \times \bigcirc$$

$$\bigcirc \times 10$$

$$2 \times \bigcirc = 20$$

$$10\sqrt{3} \div x = \sqrt{3}$$

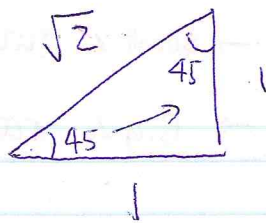
$$\frac{10\sqrt{3}}{x} = \frac{\sqrt{3}}{1}$$

$$\sqrt{3}x = 10\sqrt{3}$$

$$x = \frac{10\sqrt{3}}{\sqrt{3}} = 10$$

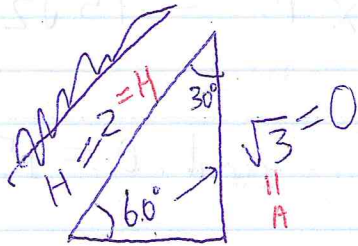
$$A : O = 1 : \sqrt{3} = x : 10\sqrt{3}$$

Example 2



$$\sin 45^\circ = \frac{O}{H} = \frac{1}{\sqrt{2}}$$

$$\cos 45^\circ = \frac{A}{H} = \frac{1}{\sqrt{2}}$$



$$\sin 30^\circ = \frac{O}{H} = \frac{1}{2}$$

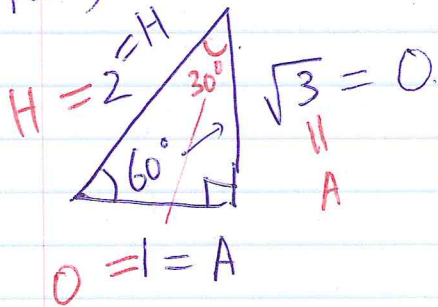
~~$$\cos 30^\circ = \frac{A}{H} = \frac{\sqrt{3}}{2}$$~~

$$A = 1 = O$$

$$\sin 60^\circ = \frac{O}{H} = \frac{\sqrt{3}}{2}$$

$$\begin{aligned} * \sqrt{2} \times \sqrt{2} &= 2^{\frac{1}{2}} \times 2^{\frac{1}{2}} = 2^{\frac{1}{2} + \frac{1}{2}} = 2^1 = 2 \\ &= \sqrt{2^2} = 2 \end{aligned}$$

Example 1)



$$\text{TOA} = \tan 60^\circ = \frac{\sqrt{3}}{1}$$

$$\text{CAH} = \cos 30^\circ = \frac{\sqrt{3}}{2}$$