MPM2D
Ms. Kueh

Factored Form of a Quadratic Relation

April 23

Recall

If the vertex is (h, k), then the equation of the **axis of symmetry** is $x = h$.

There are several different ways to write the equation of a line. What are two ways of writing the equation of a line that you learned in grade 9?

$y = mx + b$ or $ax + by + c = 0$

Definitions:

Term part of an algebraic expression, separated from the rest by plus or minus signs.

For example in $2x^2 + x - 10$, the terms are $2x^2$, x and -10

Polynomial an algebraic expression consisting of one or more terms, can be written in the form $a + bx + cx^2 + \cdots$, where a, b, and c,... are numbers.

Some examples of polynomial are: x^3, $x^2 + 1$, 7, $x^3 + x^2 + x + 5$

The degree of a one-variable polynomial is the highest exponent that appears in any term of the expanded form of the polynomial.

Example 1 Determine the degree of each polynomial

a) $2x - 2$

1

b) $3r^2$

2

c) $2x^3 - 3x^2 + x - 4$

3

d) $x^2 + 3x - 1$

2

e) $x(x + 2)$

$= x^2 + 2x$

2

f) $x^2(x^3 - 3)$

$= x^{2+3} - 3x^2$

$= x^5 - 3x^2$

$\therefore 5$

g) $x(x^2 + 2x + 1)$

$= x^3 + 2x^2 + x$

$\therefore 3$

*A quadratic relation has degree of 2.

There are 3 different ways to write a quadratic relation.

1) The **Standard Form** of a quadratic relation is $Y = ax^2 + bx + c$

2) The **Factored Form** of a quadratic relations is $Y = a(x - r)(x - s)$

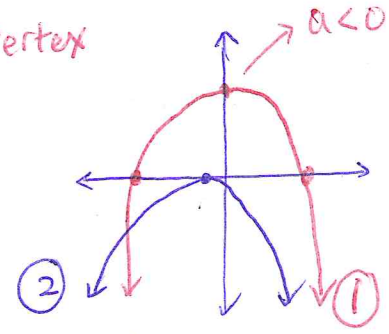
FINALLY! A reason to factor!! ☺

Direction of opening

y value of vertex

If $a > 0$, the parabola opens up and has a minimum.

If $a < 0$, the parabola opens down and has a maximum.



Number of zeros $Y = a(x-r)(x-s)$

not equal

① If $r \neq s$, then the relation has two zeros (= two x intercepts).

② If $r = s$, then the relation has one zero (= one x intercept)

also known as

Example 1 Find the zeros a.k.a. x-intercepts of the parabola described by $y = (x + 3)(x - 5)$

Zeros $\Rightarrow y = 0$

$0 = (x+3)(x-5)$ → similar to $0 = A \cdot B$ → If $A=0 \rightarrow C$ // $B=0 \rightarrow C$

$x+3=0$ or $x-5=0$

$x=-3$ $x=5$ ∴ zeros are $(-3,0)$ and $(5,0)$

$(-3,0)$ $(5,0)$

Example 2 Describe the graph of the quadratic relation $y = 2(x + 1)(x - 7)$.

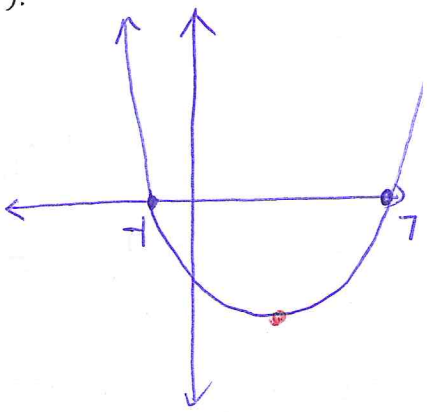
Find the

a) x-intercepts (YES, you must **show all work**) → $y=0$

$0 = 2(x+1)(x-7)$

$x+1=0$ $x-7=0$

$x=-1$ ∴ $(-1,0)$ $x=7$ ∴ $(7,0)$



b) vertex → (h,k)

$$h = \frac{x_1 + x_2}{2} = \frac{-1+7}{2} = 3$$

Sub $x=3$ into equation. ∴ Vertex = $(3,-32)$

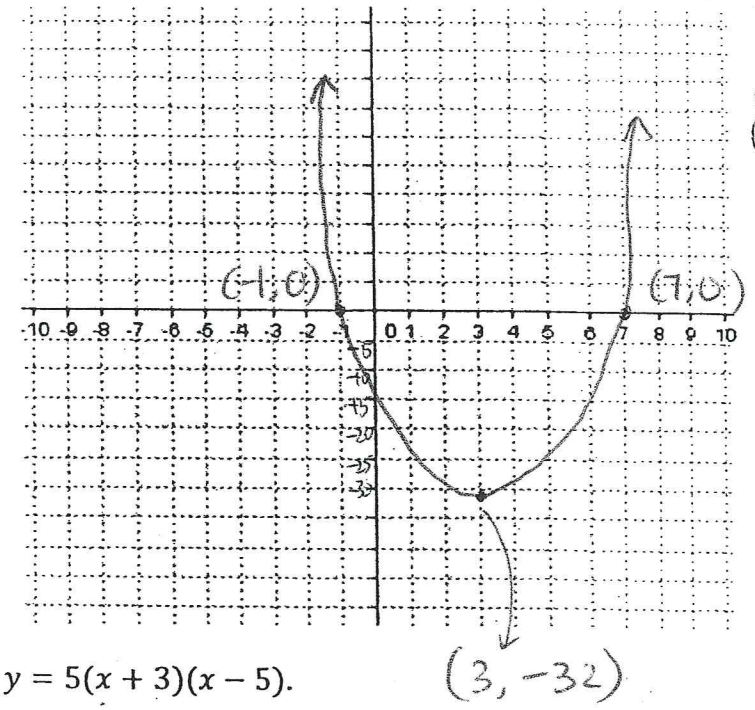
$y = 2(3+1)(3-7)$

$y = 2(4)\cdot(-4) = -32$

c) axis of symmetry

$x = 3$

d) sketch the graph



Example 3 Try it yourself: For the graph $y = 5(x + 3)(x - 5)$.
Find the

a) x-intercepts **(Show all work!)** $\rightarrow y = 0$

$0 = 5(x+3)(x-5)$

$x+3=0$ $\quad$ $x-5=0$

$x=-3$ $\quad$ $x=5$

$\therefore (-3,0)$ $\quad$ $\therefore (5,0)$

$\therefore$ Two x intercepts are $(-3,0)$ and $(5,0)$

b) vertex $\rightarrow (h,k)$

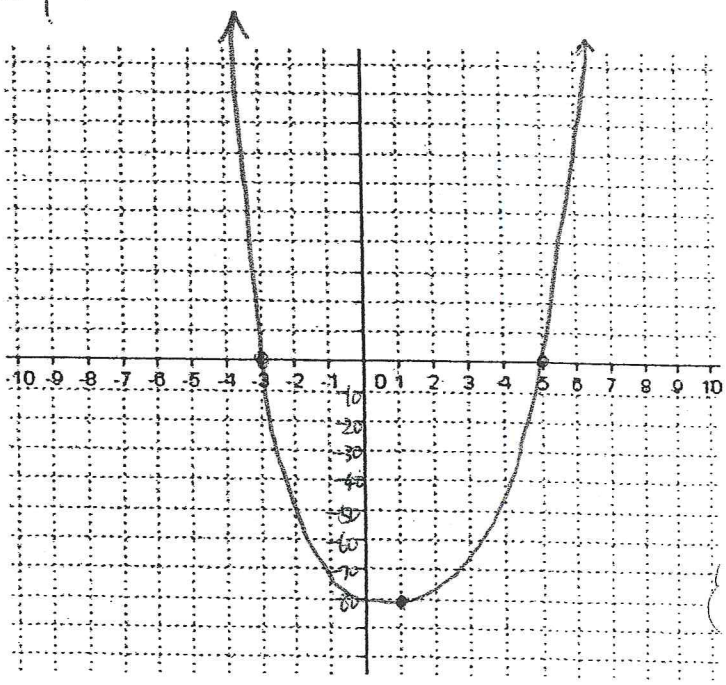
$$h = \frac{x_1+x_2}{2} = \frac{-3+5}{2} = \frac{2}{2} = 1$$

sub $x=1 \rightarrow$ equation

$y = 5(1+3)(1-5)$

$y = 5 \cdot (4)(-4)$

$y = -80$ $\quad$ $\therefore$ Vertex $= (1, -80)$



c) axis of symmetry

$x = 1$

d) sketch the graph

Homework: Worksheet