May 1st MCR3U Unit Test on Friday May 8 (might be Monday)

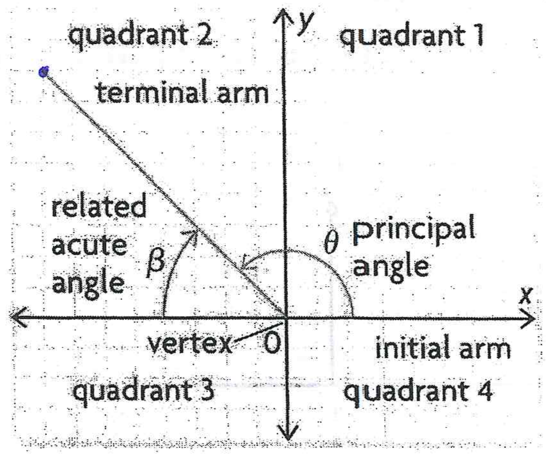
Ms. Kueh

Angles between 90° and 360°

θ = **Principal Angle** Quiz on Monday

|| Original∠

- In Standard position
- Initial arm is on positive x - axis
- is always In between 0° to 360°



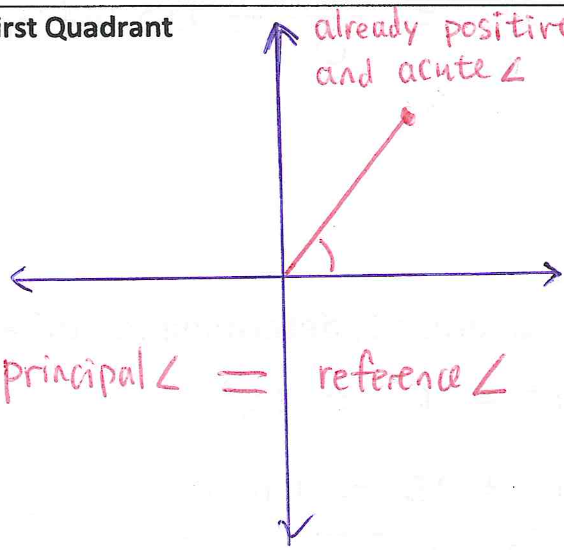
β = **Related Acute Angle/Reference Angle:**

- Positive acute angle made by the terminal arm of the angle and x -axis → less than 90°

How to find Reference angle in four quadrants:

First Quadrant

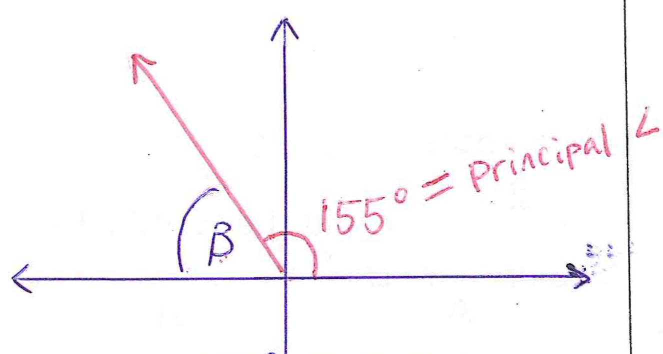
already positive and acute∠



principal∠ = reference∠

Second Quadrant

Example – Principal Angle of 155°

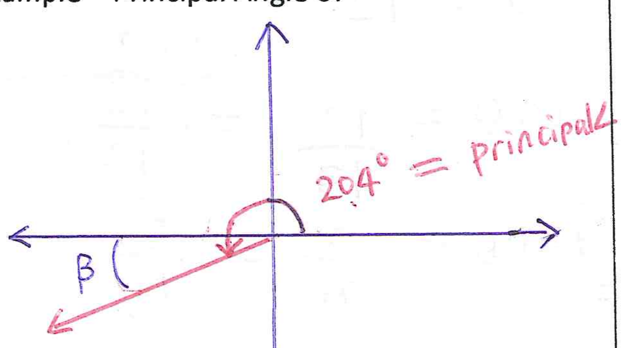


$\beta = 180 - 155° = 25°$

↳ RA∠

Third Quadrant

Example – Principal Angle of 204°

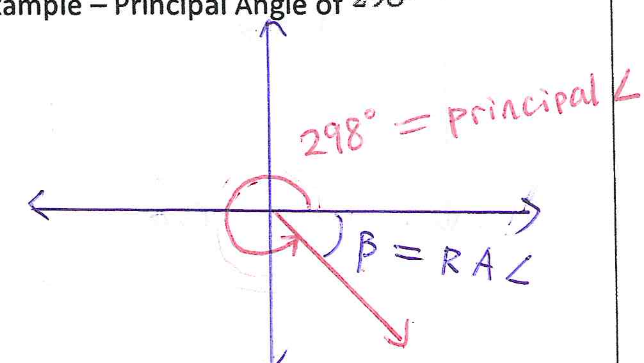


β = RA∠

↳ $= 204° - 180° = 24°$

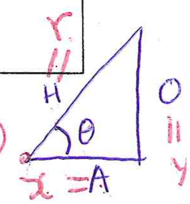
Fourth Quadrant

Example – Principal Angle of 298°

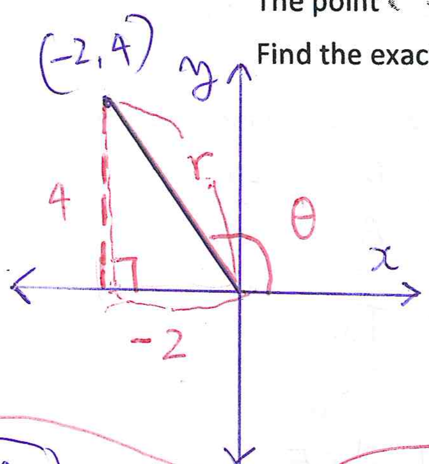


$\beta = 360° - 298° = 62°$

Example 1 Finding the trig ratios of angles between 90° and 360° given a point.



The point $(-2, 4)$ is on the terminal arm of an angle, θ in standard position. Find the exact primary trig ratios of θ .



SOH : $\sin\theta = \dfrac{y}{r}$

$r^2 = 2^2 + 4^2$

$r^2 = 4 + 16$

$\sqrt{r^2} = \sqrt{20}$

$r = \sqrt{4 \times 5}$

$\therefore r = 2\sqrt{5}$

$\sin\theta = \dfrac{\cancel{4}\,2}{\,{}_{1}\cancel{2}\sqrt{5}}$

$= \dfrac{2 \times \sqrt{5}}{\sqrt{5} \times \sqrt{5}}$

$= \dfrac{2\sqrt{5}}{5}$

CAH

$\cos\theta = \dfrac{x}{r} = \dfrac{-\cancel{2}^{\,1}}{\cancel{2}_{\,1}\sqrt{5}}$

$= \dfrac{-1 \cdot \sqrt{5}}{\sqrt{5} \cdot \sqrt{5}}$

$= \dfrac{-\sqrt{5}}{5}$

$\tan\theta = \dfrac{y}{x} = \dfrac{4}{-2}$

TOA

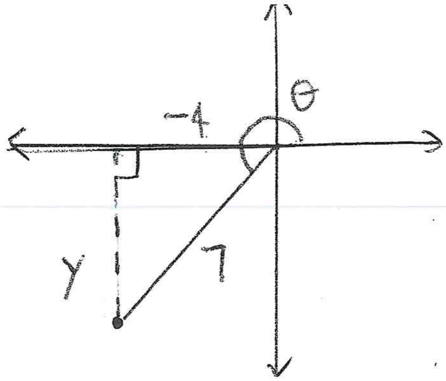
$= -2$

Example 2 Finding the trig ratios of angles between 90° and 360° given a ratio.

If $\cos\theta = -\dfrac{4}{7}$ and the terminal arm lies in quadrant 3, find the exact values of the other primary trig ratios.

$4 \rightarrow x$

$7 \rightarrow r$



$\cos\theta = \frac{x}{r} = -\frac{4}{7}$ ← x is -4 because one of the two numbers must carry ⊖ sign

$r = 7$ because r is always ⊕ #.

$y^2 + (-4)^2 = 7^2$

$y^2 = 49 - 16$

$\sqrt{y^2} = \sqrt{33}$

$\therefore y = -\sqrt{33}$ (Quadrant 3)

$\sin\theta = \frac{y}{r} = \frac{-\sqrt{33}}{7}$

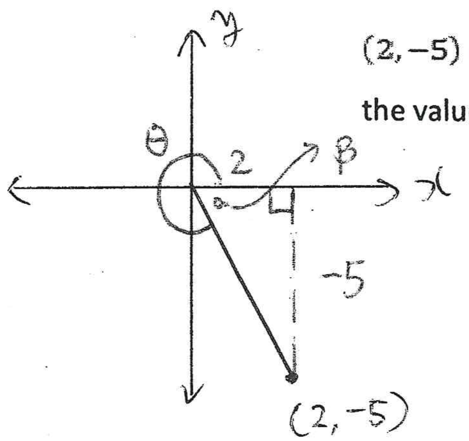
$\cos\theta = \frac{x}{r} = \frac{-4}{7}$

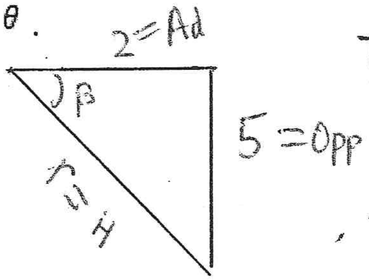
$\tan\theta = \frac{y}{x} = \frac{-\sqrt{33}}{-4}$

$\therefore \tan\theta = \frac{\sqrt{33}}{4}$

Example 3 Finding the value of an angle between 90^0 and 360^0

$(2, -5)$ lies on the terminal arm of an angle, θ in standard position. Determine the value of θ.





$\tan\beta = \frac{O}{A} = \frac{-5}{2}$

$\frac{\tan\beta}{\tan} = \frac{\frac{-5}{2}}{\tan}$

$\beta = \tan^{-1}\left(\frac{-5}{2}\right)$

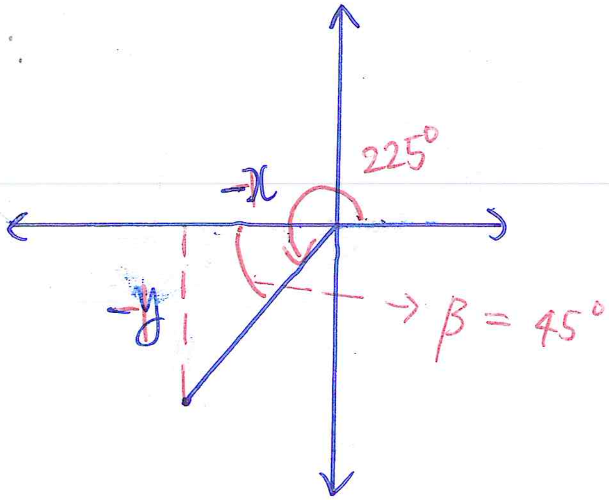
$\therefore \beta \doteq 68°$

$\theta = 360 - 68°$

$\therefore \theta = 292°$

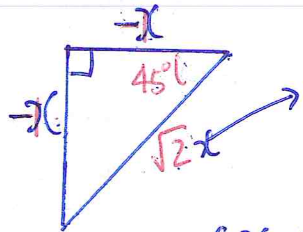
Example 4 Finding the exact value of a trig ratio of an angle between $90°$ and $360°$

Determine the exact value of $\cos 225°$.



$225° = 180° + 45°$

$\cos 225° = \cos 45°$



$\sqrt{2} \times x = \sqrt{2}x$

$\cos 45° = \frac{A}{H} = \frac{-x}{\sqrt{2}x}$

$= \frac{-1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{-\sqrt{2}}{2}$

$\therefore \cos 225° = -\frac{\sqrt{2}}{2}$

The Unit Circle

Something special happens when you look at points on the unit circle (a circle with radius 1):

Remember : principal angle's primary ratio = reference angle's ratio