May 4

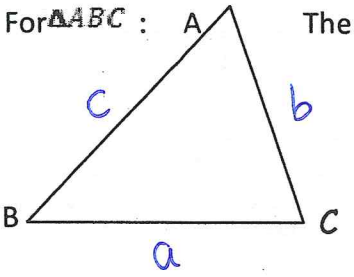
MCR3U

Ms. Kueh

# Sine and Cosine Law

The Sine and Cosine Laws are used to solve triangles that are *not right triangles*.

For $\triangle ABC$ :




The **Sine Law** states: $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$ OR $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

* You must be given at least one $\angle$ and one side which are opposite to each other.

The **Law of Cosines** states: $a^2 = b^2 + c^2 - 2bc\cos A$

$\angle A = \cos^{-1}\left(\frac{a^2 - b^2 - c^2}{-2bc}\right)$

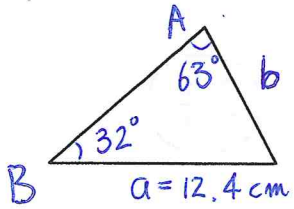
$b^2 = a^2 + c^2 - 2ac\cos B$

$c^2 = b^2 + a^2 - 2ba\cos C$

* $\angle A$ and $a$ must face each other. [ be opposite

**Example 1** Find the length of the indicated side.

a) In $\triangle ABC$: $\angle A = 63°$ , $\angle B = 32°$ , $a = 12.4\ cm$ , find $b$ .




$\frac{\sin A}{a} = \frac{\sin B}{b}$

$\frac{\sin 63°}{12.4} = \frac{\sin 32°}{b}$ ← b → unknown

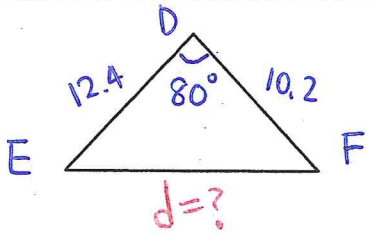
$b \cdot \sin 63° = \sin 32° \times (12.4)$

$b = \frac{\sin 32° (12.4)}{\sin 63°}$

$b = 7.375\ cm$

$\therefore b = 7.4\ cm$

b) In $\triangle DEF$: $\angle D = 80°$ , $e = 10.2\ m$ , $f = 12.4\ m$ . Find $d$ .



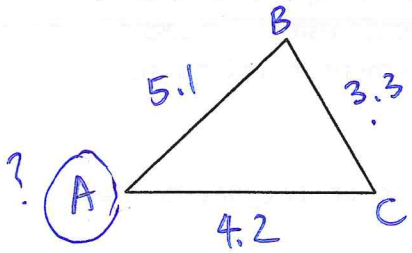

$d^2 = 12.4^2 + 10.2^2 - 2(12.4)(10.2)\cos 80°$

$d^2 = 257.8 - 43.92$

$d = 14.6\ m$

**Example 2** Find the indicated angle:

a) In $\triangle ABC$: $a = 3.3\ cm$, $b = 4.2\ cm$, $c = 5.1\ cm$. Determine the measure of $\angle A$.




$$a^2 = b^2 + c^2 - 2bc \cdot \cos A$$

$$3.3^2 = 4.2^2 + 5.1^2 - 2(4.2)(5.1) \cdot \cos A$$

$$\cos A = \frac{(a^2 - b^2 - c^2)}{(-2bc)}$$

$$A = \cos^{-1}\left(\frac{a^2 - b^2 - c^2}{-2bc}\right)$$

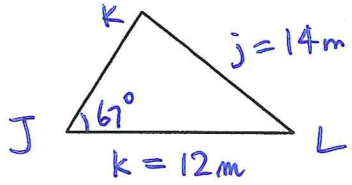
$$A = \cos^{-1}\left(\frac{3.3^2 - 4.2^2 - 5.1^2}{-2 \cdot 4.2 \cdot 5.1}\right)$$

$$A = 40.12^\circ$$

* Monday (May 4) class was finished here.

HW for today: page 221 Q 10, 11, 12, 13

b) In $\triangle JKL$: $J = 67^\circ$, $j = 14\ m$, $k = 12\ m$. Determine the measure of $\angle K$.
(Remember that if you are finding $\angle K$ using the sine law, there will be two possible answers. One works and one doesn't.)




$$\frac{j}{\sin J} = \frac{k}{\sin K}$$

$$\frac{14}{\sin 67} = \frac{12}{\sin K}$$