CAST Rule (Park) May 5 May 4 MCR3U

Thus we obtain that cosΘ and sinΘ will always be negative when constructed in quadrant 3, while tanΘ will always be positive because a negative divided by a negative is a positive.

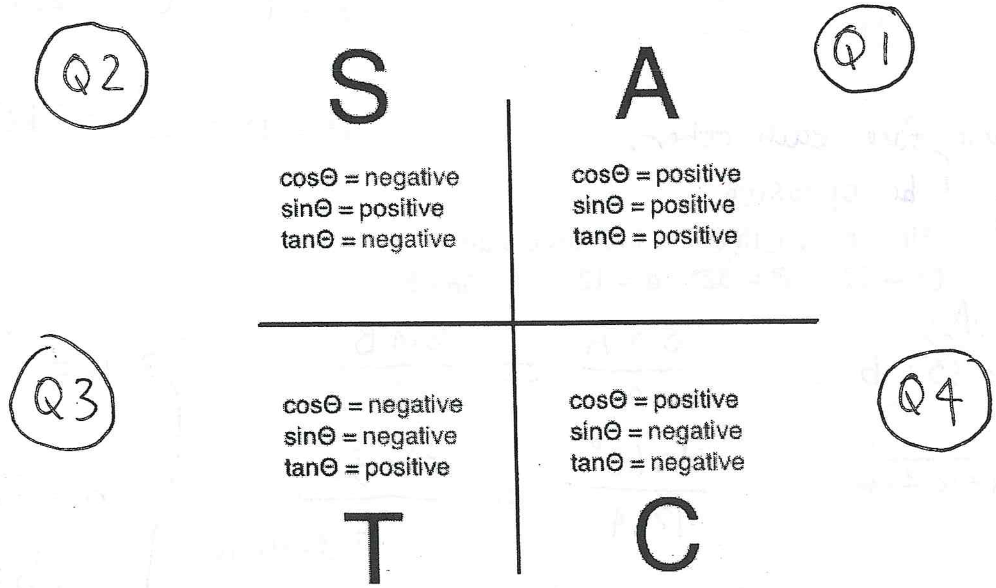
We will not go through quadrant 4, however, the procedure follows identically to that of quadrants 1, 2, and 3 we derived above. The table below will summarize the signs (positive or negative) for cosine, sine, and tangent:

	CosΘ	SinΘ	TanΘ
Quadrant 1 All	Positive	Positive	Positive
Quadrant 2 Sine	Negative	Positive	Negative
Quadrant 3 Tan	Negative	Negatives	Positive
Quadrant 4 Cos	Positive	Negatives	Negative

From this diagram, we can determine that cosΘ is positive in quadrants 1 and 4, sinΘ is positive in quadrants 1 and 2, and tanΘ is positive in quadrants 1 and 3. More intuitively, cosΘ is exclusively positive in quadrant 4, sinΘ is exclusively positive in quadrant 2, and tanΘ is exclusively positive in quadrant 3. The table below summarizes as follows:

cell-content	CosΘ	SinΘ	TanΘ
Exclusively Positive in:	Quadrant 4	Quadrant 2	Quadrant 3

Thus this is how we define the CAST Rule:



Where the letters of CAST starting at quadrant 4 going counter clockwise state which trigonometric ratios are exclusively positive in that quadrant.

Youtube : "Ambiguous Case Part 1" by Teacher Tube Math
"Ambiguous case part 2" by Teacher Tube Math
"The Cast Rule : Signs of the Ratio" by 123 Mr Bee

We can clearly see which graphs are above the x-axis (and are hence positive) on the sub-intervals of quadrant 1, quadrant 2, quadrant 3, and quadrant 4.

May 5

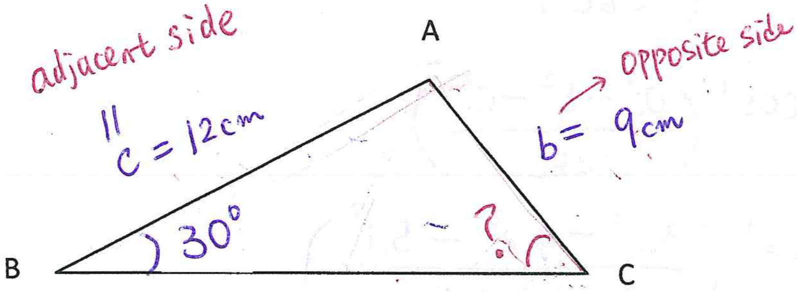
THE AMBIGUOUS CASE OF THE SINE LAW

Less than 90°

When using the Sine Law, it is sometimes not clear whether the unknown angle is acute or ~~obtuse. This occurs when you are given an angle, the adjacent and the opposite sides and the~~ opposite side is smaller than the adjacent side. If this is the case, then there are two possible triangles that can be drawn. $A > Opp$

greater than 90°

Example 3 Given △ABC, with $\angle B = 30°$, $c = 12\ cm$, $b = 9\ cm$, there are two ways to draw the triangle. $\angle C$ can be acute.



$$\frac{\sin C}{c} = \frac{\sin B}{b}$$

$$\frac{\sin C}{12} = \frac{\sin 30°}{9}$$

$$9 \sin C = 12 \times \sin 30°$$

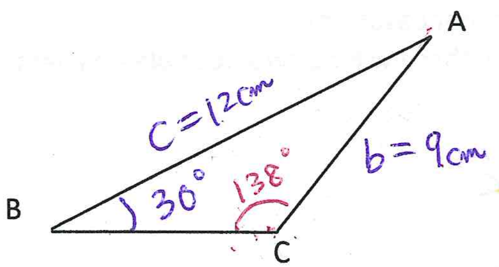
$$C = \sin^{-1}\left(\frac{12 \times \sin 30°}{9}\right)$$

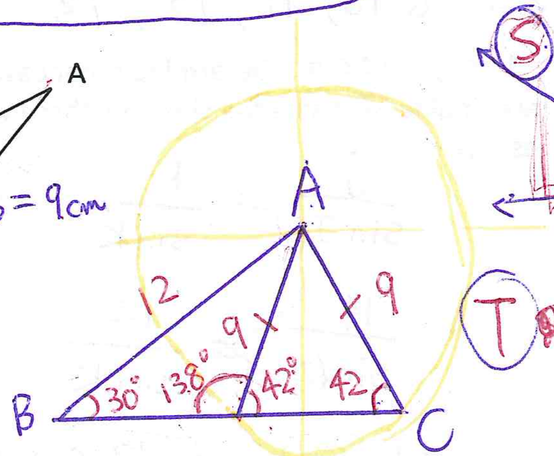
$$C = 41.8°$$

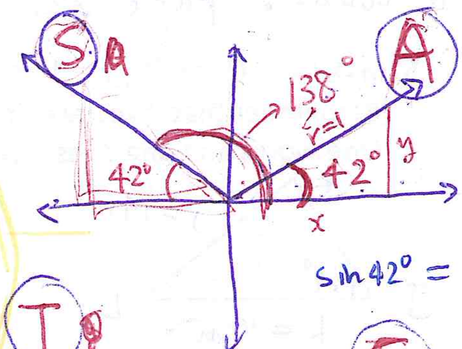
OR

∴ Both 42° and 138° are answers.

∠C can be obtuse





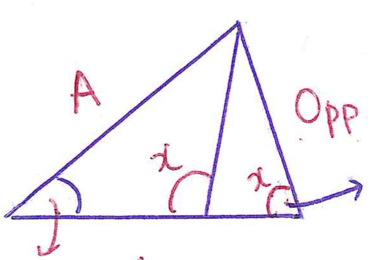


$\sin 42° = \frac{y}{r} = \frac{y}{1}$ (+)

$\sin 42° = \frac{y}{1}$ (+)

Sometimes the context of the question will make clear whether the unknown angle is obtuse or acute. In other cases, you will be required to find two different solutions to the problem.

* Ambiguous case if $A > Opp$



→ calculator's answer

acute ∠

Given angle must be

* Adjacent side = opposite from unknown angle.

Example 4 Determine whether the ambiguous case exists in each of the following:

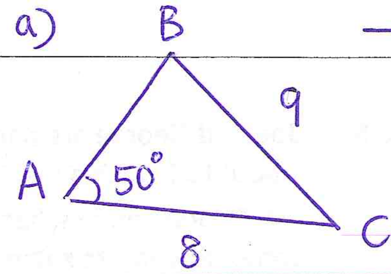
a) ΔABC, where $\angle A = 50°$, $a = 9$, $b = 8$

b) ΔABC, where $\angle A = 50°$, $a = 7$, $b = 8$

c) ΔXYZ, where $\angle Z = 37°$, $x = 17.2$, $z = 13.5$

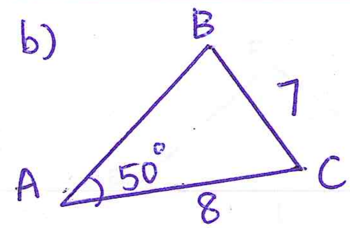
d) ΔJKL, where $\angle L = 50°$, $k = 18.7$, $j = 12.1$

a)



→ Not ambiguous case becau A < Opp

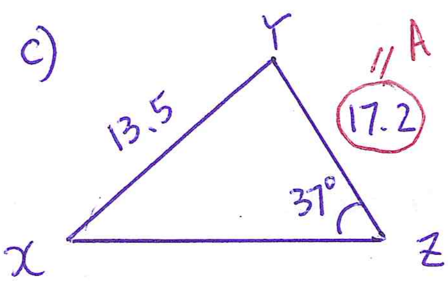
b)



Yes because A > Opp

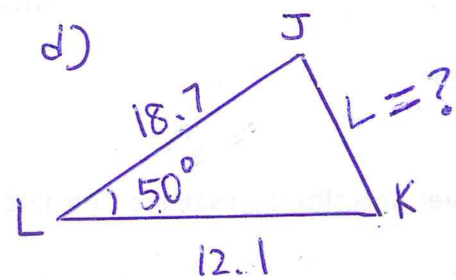
Example 5 Joe and George are part of a scientific team studying thunderclouds. The team is about to launch a weather balloon into an active part of a cloud. Joe's rope is $7.8\ m$ long and makes an angle of $36°$ with the ground. George's rope is $5.9\ m$ long. How far, to the nearest tenth of a metre, is Joe from George?

c)



Ambiguous case becauase A > opp

d)



$a^2 = b^2 + c^2 - 2bc \cdot \cos A$

$L^2 = 18.7^2 + 12.1^2 - (2 \cdot 18.7 \cdot 12.1 \cdot \cos 50$

$L^2 = 349.69 + 146.41 - 290.887$

$\sqrt{L^2} = \sqrt{205.213}$

$L = 14.3\ m$ → Not ambiguous cas because we used cosine law to find out $L = 14.3\ m$