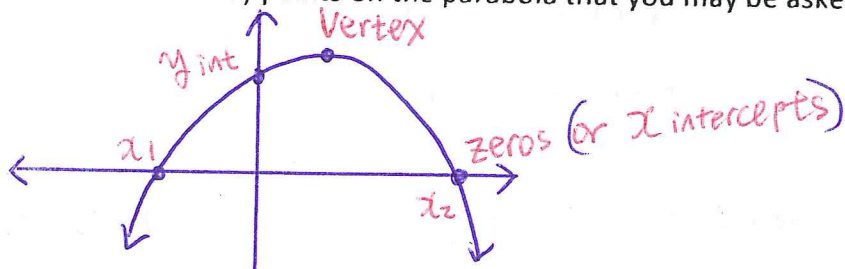When you do application problems:

Read, Understand, Do

For quadratics, there are some key points on the parabola that you may be asked to find:

1) Zeros — Sub $y=0$ and solve it for x.

2) Vertex — Max or Min point. (h, k)
— How to find Vertex? Find average of the zeros
$$h = \frac{x_1 + x_2}{2}$$
Then sub $x = h$ into eq to get y coordinate of the vertex.

3) y-intercept
Sub $x = 0 \longrightarrow$ equation

4) a point on the graph
is given, then you can sub (x, y) into the equation. Then you can find out the missing piece such as $r, s, c,$ b or a.

Example 1 A model rocket is shot into the air off a building and its path is approximated by $H(t) = -5(t + 2)(t - 6)$, where H is the height of the rocket above the ground in metres and t is the elapsed time in seconds.

a) What is the height of the building? When $t = 0$, $H = ?$

$H(0) = -5(0 + 2)(0 - 6)$
$H(0) = -5(2)(-6)$
$H(0) = 60$

∴ The height of the building is 60.

b) When will the rocket hit the ground? $\longrightarrow H(t) = 0$, $t = ?$
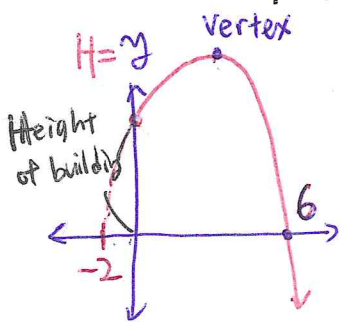
$0 = -5(t + 2)(t - 6)$

$t + 2 = 0$ $t - 6 = 0$
$t = -2$ $t = 6$

∴ The rocket will hit the ground after 6 seconds.

(We reject -2 second)

c) What is the maximum height of the rocket?

y coord of vertex

$= \dfrac{-2 + 6}{2}$

$= \dfrac{4}{2} = 2$

sub $x = 2 \rightarrow$ eq $H(t) = -5(t+2)(t-6)$

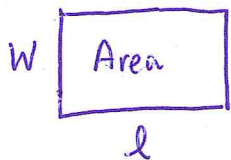$H(t) = -5(2+2)(2-6)$

$H(t) = -5(4)(-4)$

$H(t) = 80\,m$

d) What is the height of the rocket at 3 seconds? $\rightarrow t = 3, H = ?$

$H(3) = -5(3+2)(3-6)$

$H(3) = -5(5)(-3)$

$H(3) = 75$

Example 2 Wanda and Louise raise puppies. They need a rectangular fenced enclosure for the puppies to run and play in. A contractor said it will cost $30/m$ of fence, and they have $480 to spend. Find the maximum area for the puppies' play pen.

W | Area
l

Area $= l \cdot w$ (maximize) ②

$\$480 \times \dfrac{1m}{\$30} = 16m$ fence

∴ $\$480 budget means they can buy 16m fence.

Perimeter $\rightarrow P = 2l + 2w$

$\dfrac{16}{2} = \dfrac{2l}{2} + \dfrac{2w}{2}$

$8 = l + w$

$8 - l = w$ ①

Sub ① $\rightarrow$ ②
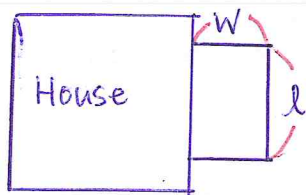
$A = l(8-l)$ (Quadratics)

$0 = l(8-l)$

$l = 0$ or 8

Vertex's $x = \dfrac{0+8}{2} = 4$

sub $x = 4 \rightarrow$ eq

$A = 4(8-4)$

$A = 16$

Example 3 Phil wants to make the largest possible rectangular vegetable garden using 18 m of fencing. The garden is right behind the back of his house, so he has to fence it on only three sides. Determine the dimensions that maximize the area of the garden.

$A = l \times W$ (Maximize) ②

$P \Rightarrow 18m = l + 2W$

$18 - 2W = l$ — ①

sub ① into ②

$A = (18 - 2W) W$ (Quadratic)

* To find zeros, sub $A = 0$

$0 = (18 - 2W) W$

$W = 0$ or $18 - 2W = 0$

$-2W = 0 - 18$

$W = \frac{-18}{-2} = 9$

* To find x coordinate of vertex

$\frac{0 + 9}{2} = \frac{9}{2} = \boxed{4.5} \ \text{//} W$

* To find y coordinate $\rightarrow$ sub $x = 4.5 \rightarrow$ eq

$A = (18 - 2 \cdot (4.5)) \ 4.5$

$A = (18 - 9) \cdot 4.5$

$A = 9 \times 4.5 = 40.5$ (max area)

When $w = 4.5 \rightarrow$ sub into ①

$l = 18 - 2(4.5)$

$l = 18 - 9 = 9$

∴ The dimensions which maximize the area are 4.5m by 9m

Worksheet

1. Michael kicks a football straight up in the air. Its height above the ground is approximated by the relation $H(t) = 25t - 5t^2$, where H is the height in metres, and t is the time in seconds.
   a. When does the football hit the ground?
   b. What are the coordinates of the vertex?
   c. Use the information you have found to graph the relation.
   d. What is the maximum height reached by the football? After how many seconds does that occur?

2. Wanda and Louise 50 m of fencing to build their puppy play pen. They want it to be rectangular using one wall of the kennel instead of fencing on one side. What is the maximum area? What dimensions give this maximum area?

3. Wanda and Louise consider instead using the corner of the kennel as 2 of the sides, so they only need to fence 2 sides. What is the maximum area if they use 50 m of fencing? What dimensions give this maximum area?

4. Wanda and Louise obviously can't make up their mind, so they also consider putting the play pen in the middle of a room, and need to fence all 4 sides. What is the maximum area if they use 50 m of fencing? What dimensions give this maximum area?

Answers:
   1.
      a. The football leaves the ground at 0 seconds and it lands at 5 seconds.
      b. The coordinates of the vertex are $(2.5, 31.25)$
      c. Sketch the graph, or check in on GeoGebra.
      d. The maximum height is 31.25 m, and it reaches that after 2.5 s.

   2. Maximum area is $312.5 \ m^2$ and dimensions are 12.5 m by 25 m

   3. Maximum area is $625 \ m^2$ and dimensions are 25 m by 25 m

   4. Maximum area is $156.25 \ m^2$ and dimensions are 12.5 m by 12.5 m