May 26

MPM2D

Ms. Kueh

Completing the Square - Applications

* Unit Test will occur on Monday (June 1st)

Recall:

1) Express the following equation in vertex form.

$y = 2x^2 + 20x + 43$ (Standard)

$y = 2(x^2 + 10x) + 43$

$a^2 + 2ab + b^2 = (a+b)^2$

$y = 2\left[x^2 + 10x + \left(\frac{10}{2}\right)^2 - \left(\frac{10}{2}\right)^2\right] + 43$

$a = x \quad b = 5$

$y = 2[(x+5)^2 - 25] + 43$

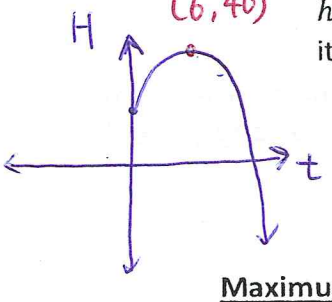
$y = 2(x+5)^2 + 43 + (-25 \times 2)$

$y = 2(x+5)^2 + 43 - 50$

$\therefore$ vertex form

$y = 2(x+5)^2 - 7$

2) A ball is thrown in the air. It's height, in metres, after t seconds is $h(t) = -5(t-6)^2 + 40$. What was the maximum height of the ball? When did it reach the maximum height?



max y value = ?

$h = 6, \quad k = 40$

Vertex $(6, 40)$ $\quad \therefore$ Max height $= 40$

Maximum/Minimum Questions

A quadratic relation reaches a maximum/minimum at Vertex.

In problems where they ask for a min or max you will be required to find vertex. To do this you want to take the equation given and put it into Vertex form form by completing the square.

Example 1 A baseball player hits a baseball into the air. The motion of the ball is modeled by the equation $h(t) = -5t^2 + 20t + 1$.

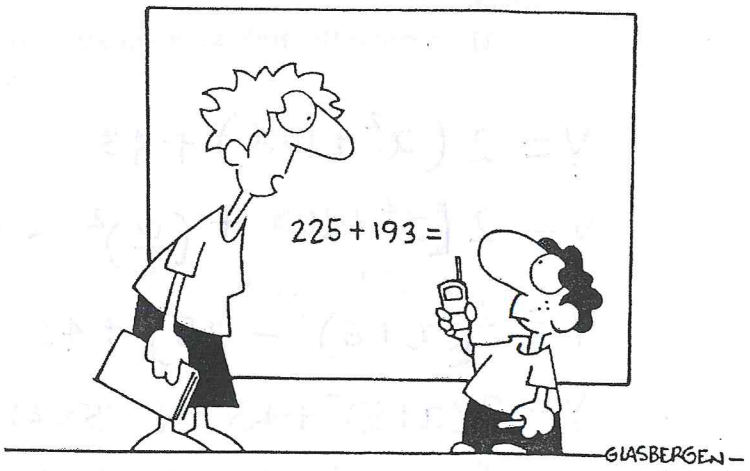
*note that -5 is rounded from -4.905, which you would use in a physics class.

a) What was the height of the ball when it was hit? time = 0 → height = ?

When $t = 0 \rightarrow \quad h(t) = -5(0)^2 + 20(0) + 1 = 1$

Completing the Square WORKSHEET

1. Write each quadratic function in the form $y = a(x - h)^2 + k$.
   a) $y = x^2 + 6x$
   b) $y = x^2 + 8x + 3$
   c) $y = x^2 - 4x - 5$
   d) $y = x^2 - 12x + 2$
   e) $y = x^2 + 14x + 39$



"You have to solve this problem by yourself. You can't call tech support."

2. Identify the vertex of each function by completing the square.
   a) $y = x^2 + 2x + 7$
   b) $y = x^2 - 24x + 215$
   c) $y = x^2 + 8x$
   d) $y = x^2 - 6x + 9$
   e) $y = 14 - 16x + x^2$

3. Determine the following for each quadratic function shown below: the direction of opening, the coordinates of the vertex, the equation of the axis of symmetry, the domain and range, and the maximum/minimum value and when it occurs.
   a) $y = -x^2 + 10x + 7$
   b) $y = 2x^2 + 12x + 65$
   c) $y = -3x^2 + 12x - 17$
   d) $y = 4x^2 + 16$
   e) $y = -7x^2 + 14x + 3$
   f) $y = -0.5x^2 + 4x - 5$
   g) $y = 5x^2 - 30x$

4. Sketch the graph of each function. Show the coordinates of the vertex, the equation of the axis of symmetry, and the coordinates of two other points on the curve.
   a) $y = x^2 - 2x - 8$
   b) $y = x^2 + 4x$
   c) $y = -x^2 - 6x - 9$
   d) $y = \frac{1}{2}x^2 + 2x + 1$
   e) $y = -3x^2 + 18x - 25$

5. Without graphing each function, state whether it has a maximum or minimum value. Determine the maximum or minimum value and when it occurs.
   a) $y = 1.5x^2 + 6x - 8$
   b) $y = 20x - 0.2x^2$
   c) $y = 2x + 1 + 0.1x^2$
   d) $y = -0.003x^2 + 0.6x - 11$

6. Sketch the graph of each function and state the coordinates of the vertex.
   a) $y = (x + 1)(x - 3)$
   b) $y = (2x + 1)(x - 2)$
   c) $y = -3(x - 1)(x + 3)$

Hint: Expand first

b) What is the maximum height of the ball?

y value of ✓ Vertex = ?

$h(t) = -5t^2 + 20t + 1$

$= -5(t^2 - 4t) + 1$

$= -5\left[t^2 - 4t + \left(\frac{-4}{2}\right)^2 - \left(\frac{-4}{2}\right)^2\right] + 1$

$a = t \quad b = -2$

$= -5[(t-2)^2 - 4] + 1$

$= -5[t-2)^2 + 1 + (-4x - 5)$

$h(t) = -5(t-2)^2 + 21$

∴ Maximum height is 21 m.

c) When does the ball reach the maximum height? x value of vertex = ?

The ball reach max height at 2 seconds.

Example 2

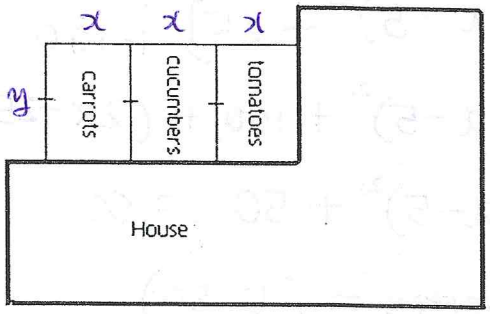
A farmer has 24 m of fence. He wants to fence a garden on the side of his L-shaped house. He doesn't need to fence the sides against the wall. His garden will have three separate sections, equal in size, one for carrots, one for tomatoes, and one for cucumbers. What is the maximum area of the whole garden?

$x + x + x + y + y + y = 24 \quad -①$

$3x + 3y = 24$

$\frac{3y}{3} = \frac{24}{3} - \frac{3x}{3}$

$y = 8 - x$



$\text{Area} = y \times 3x$

$\text{Area} = (8-x) \times 3x$

$0 = (8-x) \cdot 3x$

$8 - x = 0 \quad \text{or} \quad 3x = 0$

$x = 8 \qquad x = 0$

x value of ✓ Vertex $= \frac{(8+0)}{2} = \frac{8}{2} = 4$

Sub $x = 4$ → eq → $\text{Area} = (8-4) \times 3(4)$

$\text{Area} = 4 \times 12 = 48$

∴ The maximum area $= 48\ m^2$

What are the dimensions that maximize the fence?

When $x = 4$ → sub into $y = 8 - 4 = 4$

∴ The dimension of the fence should be 12 x 4 (width x length)

$4 \times 3 = 12$

Example 3 The sum of two numbers is 10. The sum of their squares is a minimum. Find the numbers.

① $x + y = 10 \longrightarrow y = 10 - x$

② $x^2 + y^2 =$ (Minimum) Sum of the squares

Sub ① into ②

$x^2 + (10-x)^2 = SS$ = Sum of the squares

$x^2 + 100 + x^2 - 20x = SS$

$2x^2 - 20x + 100 = SS$

standard → vertex form

$2(x^2 - 10x) + 100 =$

$2\left(x^2 - 10x + \left(\frac{-10}{2}\right)^2 - \left(\frac{-10}{2}\right)^2\right] + 100$

(a: x^2; b: $\frac{-10}{2}$)

$2[(x-5)^2 - 25] + 100$

$2(x-5)^2 + 100 + (2x - 25)$

$2(x-5)^2 + 50 = SS$

$\therefore$ Vertex $= (5, 50)$

$5^2 + y^2 = 50$

$25 + y^2 = 50$

$y^2 = 50 - 25$

$\sqrt{y^2} = \sqrt{25}$

$y = \pm 5$

so we reject -5 because $-5 + 5 = 0$

$\therefore$ The two numbers are 5 and 5

Homework: Maximum and Minimum applications problems - Height/Time and Dimension Questions, Number Problems, **TIPS Practice**