May 28 Unit test on Monday June 1st

MPM2D

Ms. Kueh

Quadratic Word Problems...All Mixed Together!

Remember, you have to read max or min (or some other derivative) in order to 'complete the square'. Otherwise you solve a quadratic equation by factoring or using the quadratic formula.

1. The cost, in dollars, of operating a machine per day is given by the formula $C(t) = 2t^2 - 84t + 1025$, where t is the time in hours, the machine operates. What is the minimum cost of running the machine? For how many hours must the machine run to reach this minimum cost?

2. A basketball is tossed from the top of a 3m wall. The height of the basketball is defined by the relation $y = -t^2 + 2t + 3$, where t represent the time in seconds, in metres, and y represents the height, in metres, above the ground.
   a) How many seconds have passed when it lands on the ground?
   b) What is the highest height the ball reaches?
   c) For how many metres horizontally is the ball higher than 3.5 m?

3. At the Mini Market at 4L bag of milk costs $3.90 and the store sells an average of 120 bags. For each $0.10 decrease in price, sales increase by 20 bags per day.
   a) Determine the price they should sell the milk at to make a revenue of $700.
   b) Determine the price they should sell the milk at to make a maximum revenue?

4. The sum of two numbers is 30 and their product is 209. Determine the two numbers.

5. The sum of two numbers is 30. Determine the two numbers if their product is a maximum.

6. The population of a town is modelled by $P(x) = -0.1x^2 + 1.2x + 4.4$, where x is the number of years since the year 2000, and P is the population in thousands.
   a) In the year 2005, what is the population?
   b) In the year 1999, what is the population?
   c) When was the population of the town the greatest? What was the greatest population?
   d) The town has really become a terrible place to live. Predict when all the residents will leave the town.

7. The product of two consecutive even integers is 224. Find the integers.

May 28 MPM2D Park

1. $C(t) = 2t^2 - 84t + 1025$ t = time in hours

C = cost

minimum cost = ?

t = ?

$$C(t) = 2(t^2 - 42t) + 1025$$
$$= 2\left(t^2 - 42t + \left(\frac{-42}{2}\right)^2 - \left(\frac{-42}{2}\right)^2\right) + 1025$$
$$= 2\left[(t-21)^2 - \frac{1764}{4}\right] + 1025$$
$$= 2(t-21)^2 + 1025 + \left(2 \cdot -\frac{1764}{4}\right)$$
$$= 2(t-21)^2 + 1025 - 882$$
$$= 2(t-21)^2 + 143 \rightarrow \text{vertex} = (21, 143)$$

∴ The minimum cost is \$143 when $t = 21$ hours

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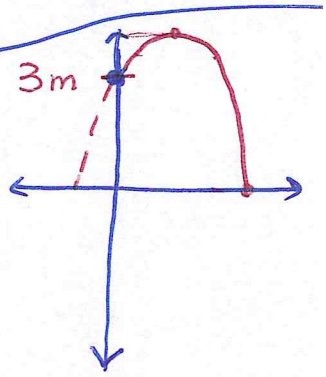
2. $y = -t^2 + 2t + 3$ t = time in seconds

y = height in metres



a) t = ? when $y = 0$

$$0 = -t^2 + 2t + 3$$

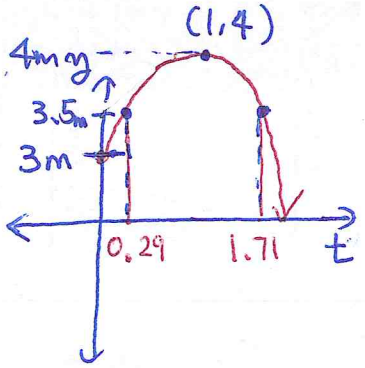
$$QF = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-2 \pm \sqrt{2^2 - 4(-1)(3)}}{2 \cdot (-1)}$$
$$= \frac{-2 \pm \sqrt{4+12}}{-2}$$
$$= \frac{-2 \pm \sqrt{16}}{-2} = \frac{-2 \pm 4}{-2}$$

$$\frac{-2+4}{-2} = \frac{2}{-2} = -1$$

$$\frac{-2-4}{-2} = \frac{-6}{-2} = 3$$

∴ It lands on the ground after 3 seconds.

2. (continued) $y = -t^2 + 2t + 3$



2b) Vertex = ?

$y = -(t^2 - 2t) + 3$

$= -(t^2 - 2t + (\frac{-2}{2})^2 - (\frac{-2}{2})^2) + 3$

$= -(t-1)^2 + 3 + (-1 \times -1)$

$= -(t-1)^2 + 3 + 1$

$= -(t-1)^2 + 4$ $\therefore$ Vertex = (1, 4)

$\therefore$ The highest height is 4m.

2c) $3.5 = -t^2 + 2t + 3$ (sub $y = 3.5$)

$0 = -t^2 + 2t + 3 - 3.5$

$0 = -t^2 + 2t - 0.5$ (standard form)

$QF = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} =$

$= \frac{-2 \pm \sqrt{2^2 - 4 \cdot -1 \cdot -0.5}}{2 \cdot -1}$

$= \frac{-2 \pm \sqrt{4 - 2}}{-2}$

$= \frac{-2 \pm \sqrt{2}}{-2}$

$= \frac{-2 \pm 1.414}{-2}$

$\frac{-2 + 1.414}{-2} = 0.293$ or

$\frac{-2 - 1.414}{-2} = 1.71$

$1.71 - 0.293 = 1.41$ seconds

$\therefore$ The ball was higher than 3.5m for 1.41 seconds.

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#3. Revenue = Unit Price x Quantity sold

Rev = $3.90 x 120

Rev = $(3.9 - 1x)(120 + 20x)$

$700 = (3.9 - x)(120 + 20x)$

$700 = 468 + 78x$

Price	Quantity
3.9	120
3.8	140
3.7	160