May 8 Unit 6 : Trig Functions

MCR3U

# Trigonometric Identities

* Unit 5 test : Monday

* Youtube : "Trigonometry : Identities" by MathTV

| Trig Equation | Trig Identity |
|---|---|
| eg. $\sin\theta = 0.6789$<br>$\frac{\sin\theta}{\sin} = \frac{0.6789}{\sin}$<br>$\theta = \sin^{-1}(0.6789)$<br>$\theta = 43°$ | eg. $\sin^2\theta + \cos^2\theta = 1$<br>e.g) $\sin^2 25° + \cos^2 25° = 1$ (Calculator!<br>e.g) $\sin^2 210° + \cos^2 210° = 1$ |

What is the difference between a trig equation and a trig identity?

Trig Eq : Looking for angle $\theta$ which makes an equation true.

Trig Identity : Mathematicians proved that this is true for all angles.

The following trig identities are important to know. They can be used to prove more complicated trig identities.

| Reciprocal Identities | Quotient Identities | Pythagorean Identities |
|---|---|---|
| $\csc\theta = \frac{1}{\sin\theta}$ | * $\tan\theta = \frac{\sin\theta}{\cos\theta}$ | * $\sin^2\theta + \cos^2\theta = 1$ |
| $\sec\theta = \frac{1}{\cos\theta}$ | $\cot\theta = \frac{\cos\theta}{\sin\theta}$ | $1 + \cot^2\theta = \csc^2\theta$ |
| $\cot\theta = \frac{1}{\tan\theta}$ | | $\tan^2\theta + 1 = \sec^2\theta$<br>$\sin^2\theta = 1 - \cos^2\theta$<br>$\cos^2\theta = 1 - \sin^2\theta$ |

To prove a trig identity, you must show that each side of the identity is equivalent.

**Some Suggestions….**

1. Start with the most complicated side and work on it until it looks like the other side or work on both sides until they look identical.
2. Rewrite all trig ratios in terms of sine and cosine.
   = Get rid off Tan, Cot, CSC, sec as much as possible.
3. If there are fractions, try a common denominator.
4. Try factoring and cancelling. *Difference of Squares often appears!

$-2^2 = -4$
$(-2)^2 = 4$

**Example 1** Prove that $1+\cot^2\theta = \csc^2\theta$. (Pretend you don't know the Pythagorean Identities)

$A = \sin\theta$

$A + A^2 = A(1+A)$

**Example 2** Prove that $\tan\theta = \dfrac{\sin\theta + \sin^2\theta}{\cos\theta(1+\sin\theta)}$.

| LS | RS |
|---|---|
| $LS = \tan\theta$ | $RS = \dfrac{\sin\theta + \sin^2\theta}{\cos\theta(1+\sin\theta)}$ |
| $= \dfrac{\sin\theta}{\cos\theta}$ | $= \dfrac{\sin\theta\cancel{(1+\sin\theta)}}{\cos\theta\cancel{(1+\sin\theta)}}$ |
| | $= \dfrac{\sin\theta}{\cos\theta}$ |

$\frac{A \cdot \cancel{B}}{C \cdot \cancel{B}}$

$\therefore LS = RS$

$\cos^2\theta + \sin^2\theta = 1 \rightarrow \sin^2\theta = 1 - \cos^2\theta$

**Example 3** Prove that $\dfrac{\sin^2\theta}{1-\cos\theta} = 1 + \cos\theta$.

| LS | RS |
|---|---|
| $LS = \dfrac{\sin^2\theta}{1-\cos\theta}$ | $RS = 1 + \cos\theta$ |
| $= \dfrac{1-\cos^2\theta}{1-\cos\theta}$ $\rightarrow a^2 - b^2 = (a+b)(a-b)$, $a = 1$, $b = \cos\theta$ | |
| $= \dfrac{(1+\cos\theta)\cancel{(1-\cos\theta)}}{\cancel{(1-\cos\theta)}}$ | $\therefore LS = RS$ |
| $= 1 + \cos\theta$ | |

**Homework:** pg. 273 #2-13, 16, 17 (Unit 6 HW)

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Ex 1) $1+\cot^2\theta = \csc^2\theta$

| LS | RS |
|---|---|
| $LS = 1+\cot^2\theta$ | $RS = \csc^2\theta$ |
| $= 1+\left(\frac{1}{\tan\theta}\right)^2$ | $= \left(\frac{1}{\sin\theta}\right)^2$ |
| $= 1+\left(\frac{\cos\theta}{\sin\theta}\right)^2$ | $= \frac{1^2}{\sin^2\theta} = \frac{1}{\sin^2\theta}$ |
| $= \frac{1}{1}+\frac{\cos^2\theta}{\sin^2\theta}$ | |
| $= \frac{\sin^2\theta}{\sin^2\theta}+\frac{\cos^2\theta}{\sin^2\theta}$ | $\therefore LS = RS$ |
| $= \frac{\sin^2\theta+\cos^2\theta}{\sin^2\theta}$ | |
| $= \frac{1}{\sin^2\theta}$ | |