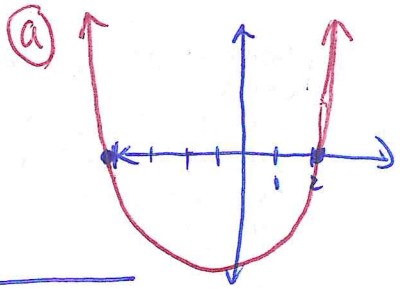
May 12

MPM2D
Ms. Kueh

Number of Roots of a Quadratic Equation

Investigation
Use the quadratic formula to find the zeros of each parabola:

a) $y = x^2 + 2x - 9$



$0 = \underset{a}{1}x^2 + \underset{b}{2}x \underset{c}{-9}$ (standard form)

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-2 \pm \sqrt{2^2 - 4 \cdot (1) \cdot (-9)}}{2 \cdot 1}$$

$$= \frac{-2 \pm \sqrt{4 + 36}}{2} = \frac{-2 \pm \sqrt{40}}{2}$$

$\frac{-2+\sqrt{40}}{2} = 2.16$

$\frac{-2-\sqrt{40}}{2} = -4.16$

∴ The two zeros are 2.16 and −4.16.

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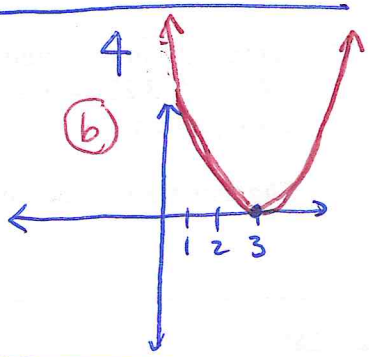
b) $y = 2x^2 - 12x + 18$

$$x = \frac{-(-12) \pm \sqrt{(-12)^2 - 4 \cdot 2 \cdot 18}}{2 \cdot 2} = \frac{12 \pm \sqrt{144 - 144}}{4}$$

$$= \frac{12 \pm 0}{4} = \frac{12}{4} = 3$$



∴ Zero is 3.

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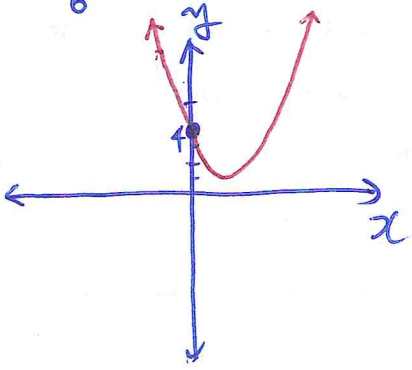
c) $y = \underset{a}{3}x^2 + \underset{c}{4}$ → $bx = 0$ → $b = 0$

$$x = \frac{-0 \pm \sqrt{0^2 - 4 \cdot 3 \cdot 4}}{2 \cdot 3} = \frac{\pm\sqrt{-48}}{6}$$



∴ Zero does not exist

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Quadratic Formula Worksheet

1. Solve using the quadratic formula. Check any three of your answers using a proper LS/RS check.
   a) $2x^2 - 3x + 1 = 0$
   b) $5x^2 - 14x - 3 = 0$
   c) $2m^2 - 5m - 12 = 0$
   d) $9p^2 - 6p + 1 = 0$
   e) $8y^2 + 6y = 9$
   f) $6g^2 = g + 2$
   g) $4x^2 - 9 = 0$

2. Solve using the quadratic formula. Express answers as exact roots and as approximate roots, rounded to the nearest hundredth. Check three of your answers using technology.
   a) $x^2 + 5x + 2 = 0$
   b) $x^2 - 3x - 1 = 0$
   c) $x^2 - x - 3 = 0$
   d) $2x^2 + 3x - 7 = 0$
   e) $m^2 - 5m = 2$
   f) $3b^2 - b = 1$

3. Solve. Express answers as integers or as decimals, to the nearest tenth.
   a) $5c^2 = 8c$
   b) $\frac{x^2}{2} - x - \frac{5}{2} = 0$
   c) $2w(w - 3) = 7$
   d) $(m - 4)(m - 2) = 12$
   e) $(6y - 1)(y + 5) = 15y - 9$
   f) $2(x + 1)(x - 2) - (x + 3) = 0$
   g) $(2d + 3)(d - 2) = (d + 9)(d - 3) + 16$

4. Solve. Round answers to the nearest hundredth.
   a) $0.1x^2 + 0.4x - 0.3 = 0$
   b) $0.25h^2 - h - 1.5 = 0$
   c) $1.2n^2 = 1.4n + 1$
   d) $-0.2 = -4.4v^2 - 4.8v$

5. A recently constructed bridge in British Columbia is modelled by the equation $-0.04x^2 + 3.28x = 0$. Find the horizontal distance, in metres, across this bridge by solving the equation.

6. *Challenge:* Solve the following equation for x: $x^2 + 2xy - y^2 = 0$

Answers:

1. a) ½, 1 b) -1/5, 3 c) -3/2, 4 d) 1/3 e) -3/2, ¾ f) -1/2, 2/3 g) -3/2, 3/2

2. a) $= \frac{-5 \pm \sqrt{17}}{2}$; $x = -0.44, -4.56$ b) $x = \frac{3 \pm \sqrt{13}}{2}$; $x = 3.30, 0.30$ c) $x = \frac{1 \pm \sqrt{13}}{2}$; $x = 2.30, -1.30$

   d) $x = \frac{-3 \pm \sqrt{65}}{4}$; $x = 1.27, -2.77$ e) $m = \frac{5 \pm \sqrt{33}}{2}$; $m = 5.37, -0.37$ f) $b = \frac{1 \pm \sqrt{13}}{6}$; $b = 0.77, -0.43$

3. a) c = 0, 1.6 b) x = 3.4, =1.4 c) w = 3.9, -0.9 d) m = 6.6, -0.6 e) y = -0.3, -2 f) x = 2.8, -1.3 g) d = 6.2, 0.8

4. a) x = 0.65, -4.65 b) h = 5.16, -1.16 c) n = 1.67, -0.5 d) v = 0.32, -1.41

5. 82 m

6. $x = (-1 \pm \sqrt{2})y$

1. Graph the following parabolas on the grid below using any method from the last unit.

a) $y = x^2 + 2x - 9$

* x intercepts are 2.16 and -4.16
* y int = -9
* vertex = $\frac{(2.16 - 4.16)}{2}$

$= \frac{-2}{2} = -1$

* $x = -1 \rightarrow$ eq

$y = (-1)^2 + 2(-1) - 9$

$y = 1 - 2 - 9$

$y = -10 \rightarrow$ vertex $(-1, -10)$

b) $y = 2x^2 - 12x + 18$

* Factoring method

$0 = 2x^2 - 12x + 18$

$0 = 2(x^2 - 6x + 9)$ [$a = x$, $b = 3$

$a^2 - 2ab + b^2 \rightarrow (a-b)^2$

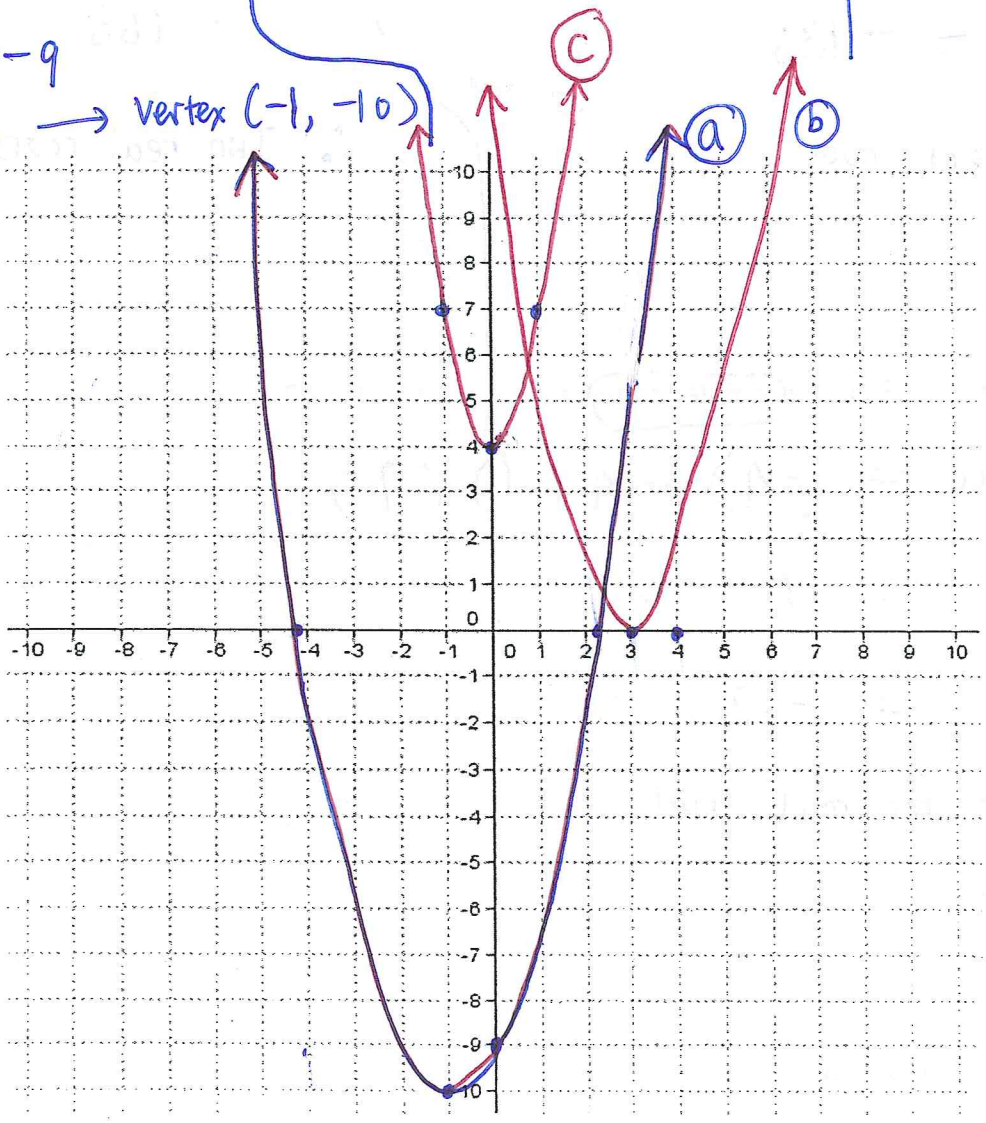
$0 = 2(x-3)^2$

$\therefore x$ int $= 3$

y int $= 18$

c) $y = 3x^2 + 4$ = y int

x	y
0	4
1	7
2	16
-1	7



$\frac{-b \pm \sqrt{b^2-4ac}}{2a}$

Definition: b^2-4ac is called the discriminant. It tells us how many roots a quadratic equation will have.

If the b^2-4ac is **positive**, there will be two real roots.

If the b^2-4ac is **zero**, there will be one real root.

If the b^2-4ac is **negative**, there will be no real root.

Example 1 Determine the number of roots each equation

a) $-5x^2+8x-10=0$ ($a=-5$, $b=8$, $c=-10$)

$b^2-4ac = 8^2-4(-5)(-10)$

$= 64-200$ (odd # of ⊖ # → ⊖ answer)

$= -136$

∴ No real root.

b) $7x^2-6=0$ ($a=7$, $b=0$, $c=-6$)

$b^2-4ac = 0^2-4(7)(-6)$

$= 0+168$ (even # of ⊖ # → ⊕ answer)

$= 168$

∴ Two real roots

Example 2 Find the complex roots of: $y = x^2-4x+7$

$b^2-4ac = (-4)^2-4(1)(7)$

$= 16-28$

$= -12$

∴ No real roots

Homework: Worksheet