May 13

MPM2D
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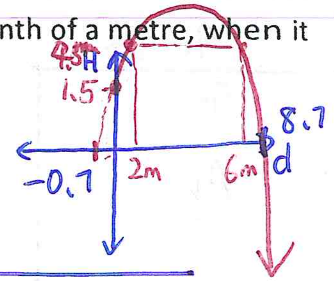
Quadratic Formula Applications

Example 1 The path of a basketball after it is thrown from a height of 1.5 m above the ground is given by the equation $H(d) = -0.25d^2 + 2d + 1.5$ where H is the height, in metres, and d is the horizontal distance, in metres.

a) How far has the ball travelled horizontally, to the nearest tenth of a metre, when it lands on the ground?

When $H = 0 \rightarrow d = ?$



$$0 = \underset{a}{\underset{\|}{-0.25}}d^2 + \underset{b}{\underset{\|}{2}}d + \underset{c}{\underset{\|}{1.5}}$$

$$QF = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-2 \pm \sqrt{2^2 - 4(-0.25)(1.5)}}{2 \cdot (-0.25)}$$

$$= \frac{-2 \pm \sqrt{4+1.5}}{-0.5} = \frac{-2 \pm \sqrt{5.5}}{-0.5} = \frac{-2 \pm 2.345}{-0.5}$$

$$x_1 = \frac{-2 + 2.345}{-0.5} = -0.69 = -0.7$$

$$x_2 = \frac{-2 - 2.345}{-0.5} = +8.69 = +8.7$$

$\therefore$ The ball travelled horizontally 8.7m. (= reject −0.7)

b) Find the horizontal distance when the basketball is at a height of 4.5 m above the ground.

When $H = 4.5m \rightarrow d = ?$

$$4.5 = -0.25d^2 + 2d + 1.5$$

$$0 = -0.25d^2 + 2d + 1.5 - 4.5$$

$$0 = \underset{a}{\underset{\|}{-0.25}}d^2 + \underset{b}{\underset{\|}{2}}d \underset{c}{\underset{\|}{-3}}$$

$\therefore$ At the height of 4.5m, the ball travel[led] 2m or 6m horizont[ally]

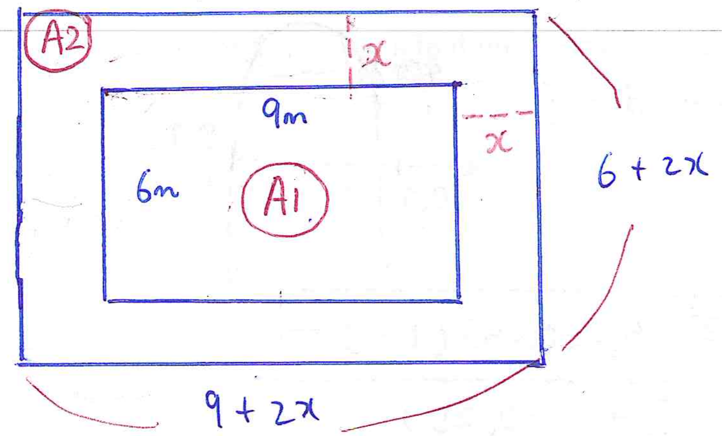
$$QF: x = \frac{-2 \pm \sqrt{2^2 - 4(-0.25)(-3)}}{2 \cdot (-0.25)} = \frac{-2 \pm \sqrt{4-3}}{-0.5} =$$

$$= \frac{-2 \pm \sqrt{1}}{-0.5} = \frac{-2 \pm 1}{-0.5} \Rightarrow \begin{cases} x_1 = \frac{-2+1}{-0.5} = \frac{-1}{-0.5} = 2 \\ x_2 = \frac{-2-1}{-0.5} = \frac{-3}{-0.5} = 6 \end{cases}$$

Example 2 Width of a Path

The parks department is planning a new flower bed outside city hall. It will be rectangular with dimensions 9 m by 6 m. The flower bed will be surrounded by a path of constant width with the same area as the flower bed.

a) Calculate the width of the path.



$A1 = 9m \times 6m = 54m$

$A2 = A1 = 54m$

$\therefore$ Total Area $= 54 + 54 = 108m$

* Equation:

$108 = (6+2x)(9+2x)$

$108 = 54 + 12x + 18x + 4x^2$

$0 = 4x^2 + 30x + 54 - 108$

$0 = 4x^2 + 30x - 54$

$a = 4,\ b = 30,\ c = -54$

QF: $x = \dfrac{-30 \pm \sqrt{30^2 - 4 \cdot 4 \cdot (-54)}}{2 \cdot 4}$

$x = \dfrac{-30 \pm \sqrt{900 + 864}}{8}$

$x = \dfrac{-30 \pm \sqrt{1764}}{8}$

$x = \dfrac{-30 \pm 42}{8}$

$x_1 = \dfrac{-30 + 42}{8} = \dfrac{12}{8} = \dfrac{3}{2}$

$= 1.5$

$x_2 = \dfrac{-30 - 42}{8} = \dfrac{-72}{8} =$

$= -9$ (reject this one b/c you can't have ⊖ width.)

$\therefore$ path $= 1.5m$

ⓐ's answer

ⓑ

$\therefore$ When $x = 1.5 \rightarrow 6 + 2(1.5)$

$= 9 \rightarrow$ length

$x = 1.5 \rightarrow$ width: $9 + 2(1.5)$

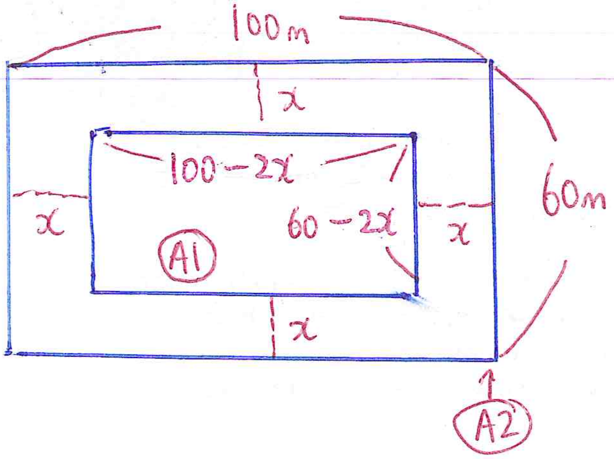
$= 12 \rightarrow$ width

$\therefore$ Perimeter of the flower bed path is 9m and 12m.

b) Calculate the perimeter of the outside of the path.

Example 3 Width of a Path

A rectangular park measures 100 m by 60 m. A path of uniform width is to be paved around the perimeter. The mayor wants to be sure the path does not reduce the area of grass by more than 10%. What is the maximum allowable width of the path, rounded to the nearest tenth of a metre?



Total Area (A2) $= 60 \times 100 = 6000\,m^2$

Mayor's grass area $= 6000 \times 0.9 = 5400\,m^2$

(A1) $= 5400 = (100-2x)\cdot(60-2x)$

$5400 = 6000 - 200x - 120x + 4x^2$

$0 = 6000 - 5400 - 320x + 4x^2$

$0 = \underbrace{600}_{=c} \underbrace{-320}_{=b}x + \underbrace{4}_{=a}x^2$

∴ Maximum path = 1.9m

QF: $x = -(-320) \pm \sqrt{(-320)^2 - 4(4)(}$

$x = \dfrac{320 \pm \sqrt{102400 - 9600}}{2 \cdot 4}$

$x = \dfrac{320 \pm \sqrt{92800}}{8}$

$x = \dfrac{320 \pm 304.6}{8}$

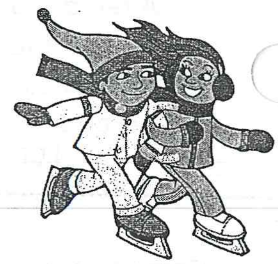
$x_1 = \dfrac{320 + 304.6}{8} = 78.1$ reject it because 78 m > 60 m

$x_2 = \dfrac{320 - 304.6}{8} = 1.9$ ✓

Homework: Worksheet

∴ Max path = 1.9m.

Quadratic Formula Applications Worksheet



1. The hypotenuse of a right triangle measures 20 cm. The sum of the lengths of the other two sides is 28 cm. Find the lengths of these two sides.

2. A rectangular skating rink measures 40 m by 20 m. It is to be doubled in area by extending each side by the same amount. Determine the new dimensions, to the nearest tenth of a metre.

3. A triangle has a height of 6 cm and a base of 8 cm. If the height and the base are both decreased by the same amount, the area of the new triangle is 20 cm^2. What are the base and height of the new triangle, to the nearest tenth of a centimetre.

4. The size of a television screen or a computer monitor is usually stated as the length of the diagonal. A screen has a 38-cm diagonal. The width of the screen is 6 cm more than the height. Find the dimensions of the screen, to the nearest tenth of a centimetre.
5. Determine the side length of a square, to the nearest hundredth of a centimetre, that has the same area as a circle of radius 10 cm.

6. The height of a triangle is 2 units more than the base. The area of the triangle is 10 square units. Find the base, to the nearest hundredth.

7. A cylinder has a height of 5 cm and a surface area of 100 cm^2. Find the radius of the cylinder, to the nearest tenth of a centimetre.

8. A sporting goods store sells 90 ski jackets in a season for $200 each. Each $10 decrease in the price would result in five more jackets being sold.
   a) Find the number of jackets sold and the selling price to give revenues of $17 600 from sales of ski jackets.
   b) What is the lowest price that would produce revenues of at least $15 600? How many jackets would be sold at this price?

Answers:

1. 12 cm, 16 cm
2. 51.2m by 31.2m
3. b = 7.4 cm, h = 5.4 cm
4. h = 23.7 cm, w = 29.7 cm
5. 17.72 cm
6. 3.58 units
7. 2.2 cm
8. a) 110 jackets at $160 each OR 80 jackets sold at $220 each b) 130 jackets at $120 each