May 14 Park

MPM2D
Ms. Kueh

More Quadratic Formula Applications

Example 1 Model the path of a Toy Rocket

The formula $H(t) = -\frac{1}{2}gt^2 + v_0t + h_0$ can be used to model the height of a projectile, where g is acceleration due to gravity, which is 9.8 m/s^2 on Earth, v_0 is the initial vertical velocity, in metres per second, and h_0 is the initial height, in metres.

a) Create a model for the height of a toy rocket launched upward at 60 m/s from the top of a 3 m platform. ⇒ initial height or $h_0 = 3m$

Initial speed or $V_0 = 60 m/s$

$H(t) = -\frac{1}{2}(9.8)t^2 + 60t + 3$

or

$H(t) = -4.9t^2 + 60t + 3$

b) How long would the rocket take to fall to Earth, rounded to the nearest hundredth of a second?

When $H(t) = 0 \rightarrow t = ?$

$0 = -4.9t^2 + 60t + 3$ ($a = -4.9$, $b = 60$, $c = 3$)

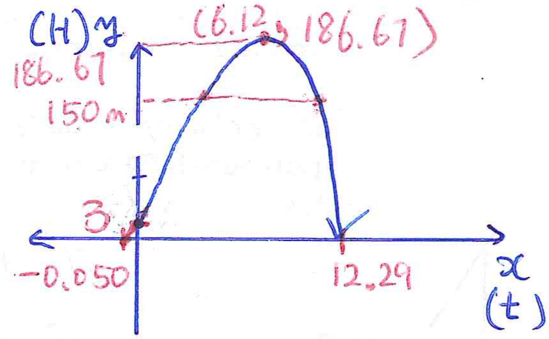
$QF = \frac{-60 \pm \sqrt{60^2 - 4(-4.9)(3)}}{2 \cdot (-4.9)}$

$QF = \frac{-60 \pm \sqrt{3600 + 58.8}}{-9.8}$

$QF = \frac{-60 \pm 60.4880}{-9.8}$ ⇒ $x_1 = \frac{-60 + 60.4880}{-9.8} = -0.0498 = -0.050$ (reject because you can not have negative time)

$x_2 = \frac{-60 - 60.4880}{-9.8} = 12.29$

∴ It takes 12.29 seconds to fall to Earth.



c) What is the maximum height of the rocket, rounded to the nearest metre?

= max y value? vertex?

* Vertex's x value $= \frac{x_1 + x_2}{2} = \frac{-0.050 + 12.29}{2} = 6.12$

* Sub $x = 6.12$ into equation

$H = -4.9(6.12)^2 + 60(6.12) + 3$

$= -183.526 + 367.2 + 3 = 186.67$

∴ The max height of the rocket is 186.67 m.

d) Over what time interval is the height of the toy rocket greater than 150 m? Round to the nearest hundredth of a second.

Sub $y = 150 \rightarrow$ eq

$150 = -4.9t^2 + 60t + 3$

$0 = -4.9t^2 + 60t + 3 - 150$

$0 = -4.9t^2 + 60t - 147$

$=a \quad \| b \quad \| c$

$$QF = \frac{-60 \pm \sqrt{60^2 - 4 \cdot (-4.9)(-147)}}{2 \cdot (-4.9)}$$

$$QF = \frac{-60 \pm \sqrt{3600 - 2881.2}}{-9.8}$$

$$QF = \frac{-60 \pm 26.8104}{-9.8}$$

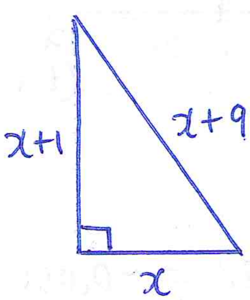
$$x_1 = \frac{-60 + 26.8104}{-9.8} = 3.39$$

$$x_2 = \frac{-60 - 26.8104}{-9.8} = 8.86$$

$\therefore$ The height of rocket is greater than 150m in between $t = 3.39$ s and $t = 8.86$ second.

Example 2 Right Triangle

One leg of a right triangle is 1 cm longer than the other leg. The length of the hypotenuse is 9 cm greater than that of the shorter leg. Find the lengths of the three sides.



$x^2 + (x+1)^2 = (x+9)^2$ $* (a+b)^2 = a^2 + b^2 + 2ab$

$a^2 + b^2 = c^2$ (Pythagorian theorem)

$x^2 + x^2 + 1^2 + 2x = x^2 + 81 + 18x$

$2x^2 - x^2 + 2x - 18x + 1 - 81 = 0$

$x^2 - 16x - 80 = 0$ $ac = 1 \times -80 = -80$

$(x-20)(x+4) = 0$ $b = -16$

$-20 \times 4 = -80$

$x = 20$ and ~~-4~~ $-20 + 4 = -16$

↳ you can't have negative side.

$\therefore$ The three sides are 20 cm, 21 cm, 29 cm.

Homework: Worksheet