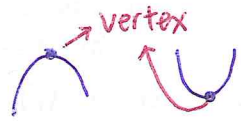
MPM2D
Ms. Kueh

May 15

Vertex Form of a Quadratic Relation



Recall: 1) (h, k) is the vertex of the graph.

2) The standard form of a quadratic relation is: $f(x) = ax^2 + bx + c$

3) The factored form of a quadratic relation is: $f(x) = a(x-r)(x-s)$

Another way to express a quadratic relation is the vertex form:

$f(x) = a(x-h)^2 + k$

→ a is the same in factored form and standard form
→ h and k are the coordinates of the vertex. (h, k)

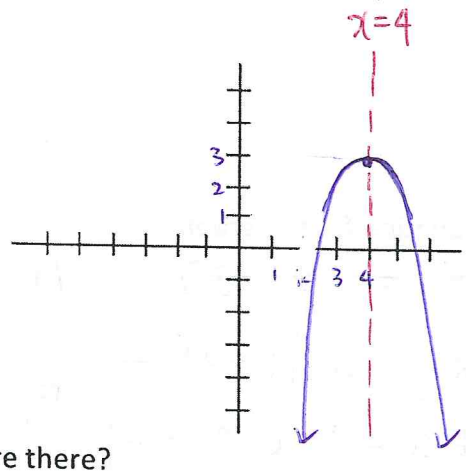
Example 1 For the equation $f(x) = -2(x-4)^2 + 3$

a) What is the vertex of the graph of the quadratic relation? $(4, 3)$

b) What is the axis of symmetry? $x = 4$

c) What is the direction of opening? $-2 \rightarrow \cap \rightarrow$ opens down

d) Sketch the graph:



e) How many zeros are there? 2

f) Find the standard form of the quadratic relation.

$f(x) = -2(x-4)^2 + 3$
$= -2(x-4)(x-4) + 3$
$= -2(x^2 - 4x - 4x + 16) + 3$
$= -2(x^2 - 8x + 16) + 3$
$= -2x^2 + 16x - 32 + 3$
$= -2x^2 + 16x - 29$

∴ standard form is $f(x) = -2x^2 + 16x - 29$

Example 2 Find the equation of the parabola in vertex form if

a) $a = 5$, and the vertex is at $(-2, 6)$

$f(x) = a(x-h)^2 + k$

$f(x) = 5(x+2)^2 + 6$

b) $a = 3$, vertex at $(-4, -5)$

$f(x) = 3(x+4)^2 - 5$

Example 3

Find the equation in vertex form if a parabola has vertex at $(5, -1)$, and it passes through $(4, 1)$.

$h = 5$ $\quad k = -1$

$f(x) = a(x-5)^2 - 1$

sub $x = 4$, $y = 1$

$1 = a(4-5)^2 - 1$

$1 = a(-1)^2 - 1$

$1 = a - 1$

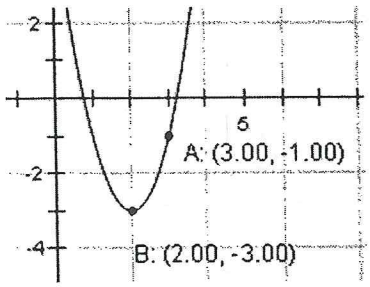
$1 + 1 = a$

$2 = a$

$\therefore$ The vertex form is:

$f(x) = 2(x-5)^2 - 1$

Example 4 Find the vertex form equation for this graph:



Vertex $= (2, -3)$ $\quad A(3, -1)$

$f(x) = a(x-2)^2 - 3$

sub $x = 3$, $y = -1$

$-1 = a(3-2)^2 - 3$

$-1 = a(1)^2 - 3$

$-1 = a - 3$

$-1 + 3 = a$

$2 = a$

$\therefore$ Vertex form is

$f(x) = 2(x-2)^2 - 3$

Example 5 Find the vertex form equation of $f(x) = 3(x-1)(x+2)$.

Since x intercepts are 1 and $-2 \rightarrow \frac{1+(-2)}{2} = \frac{-1}{2}$

* Vertex's x coordinate $= -\frac{1}{2}$ sub this into equation

$$f(x) = 3\left(-\tfrac{1}{2}-1\right)\left(-\tfrac{1}{2}+2\right)$$
$$= 3\left(-\tfrac{1}{2}-\tfrac{2}{2}\right)\left(-\tfrac{1}{2}+\tfrac{4}{2}\right)$$
$$= 3\left(-\tfrac{3}{2}\right)\left(\tfrac{3}{2}\right)$$
$$= \frac{-27}{4}$$

$\therefore$ Vertex form is $f(x) = 3\left(x+\frac{1}{2}\right)^2 - \frac{27}{4}$

Example 6 A ball is hit into the air. Its height H (in metres) after t seconds is $H(t) = -5(t-4)^2 + 120$. (= vertex form $\therefore$ 120 is not y intercept)

a) Which direction does the parabola open? Does this make sense?

$a = -5 \rightarrow \cap \rightarrow$ open down Yes it makes sense.

b) What are the coordinates of the vertex? What does the vertex represent in this situation?

Vertex is $(4, 120)$. The vertex represents the maximum height of the ball.

c) From what height was the ball hit?

Initial height? y intercept $= ?$ $\rightarrow$ sub $x=0$ or $t=0$

$$H(t) = -5(0-4)^2 + 120$$
$$H(t) = -80 + 120$$
$$H(t) = 40\text{m}$$

$\therefore$ The ball was initially hit from 40 m cliff. (or building.)

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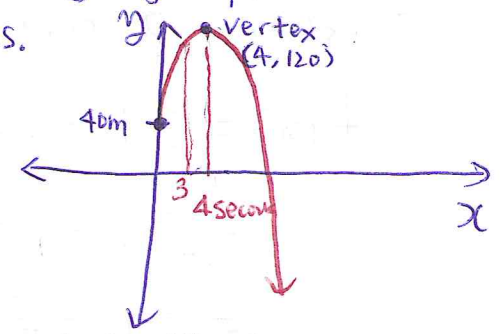
d) Is the ball going up or down at $t = 3$? Explain.

Since intial height is 40 m and vertex is (4, 120)

So we can reasonably assume that the ball is going up at $t = 3$ seconds.



e) Is the ball still in the air after 9 s? Explain.

sub $t = 9$ → equation

$H(t) = -5(9-4)^2 + 120$

$H(t) = -5 \cdot 25 + 120$

$H = -125 + 120 = -5$

∴ The ball is not in the air after 9 seconds.

Example 7 Find the vertex form equation for a parabola with zeros at -4 and 2 and a y-intercept of -24.

→ $(0, -24)$ (x y)

x intercepts

$y = a(x-r)(x-s)$

$y = a(x+4)(x-2)$

$-24 = a(0+4)(0-2)$

$-24 = a \cdot (4) \cdot (-2)$

$\frac{-24}{-8} = \frac{-8a}{-8}$

$3 = a$

∴ $y = 3(x+4)(x-2)$.

→ $y = a(x-h)^2 + k$

* Vertex $= \frac{x_1 + x_2}{2} = \frac{-4+2}{2} = -1$

x coord

sub $x = -1$ into eq

$y = 3(-1+4)(-1-2)$

$y = 3(3)(-3) = -27$

∴ Vertex $= (-1, -27)$

∴ $y = 3(x+1)^2 - 27$

Something to think about: In the equation $y = a(x-h)^2 + k$, How does a, h, and k change the graph of $y = x^2$? Why?

Homework: Textbook pg. 351 #2a-c, 3cd, 4, 5~~cd, 6b-d, 7ac,~~ 8ac, 9bde, 10ad, 13, 20, 21a-d, 24

Friday's HW

Monday's HW