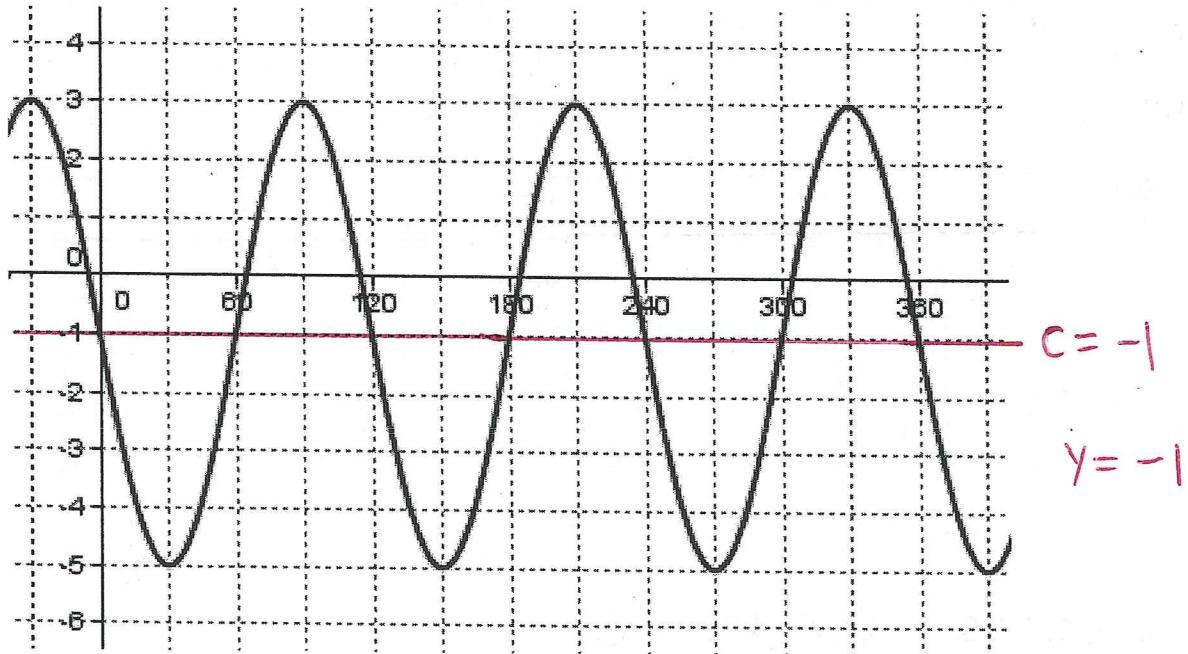


Finding the Equation from a Graph

Example 1

- a) Find the equation of the graph below using the base function $y = \sin x$.



$$* y = a \sin(k(x-d)) + c$$

$$* c = \text{axis or middle line} = \frac{3 + (-5)}{2} = -1 \therefore c = -1$$

* a = vertical distance from top to bottom then divide by 2

$$\frac{3+5}{2} = \frac{8}{2} = 4 \quad \therefore a = 4 \quad \text{or } \frac{3-(-5)}{2}$$

$$* \text{period} = \frac{360}{k} = \text{horizontal distance between two crests} \Rightarrow 210 - 90 = 120$$

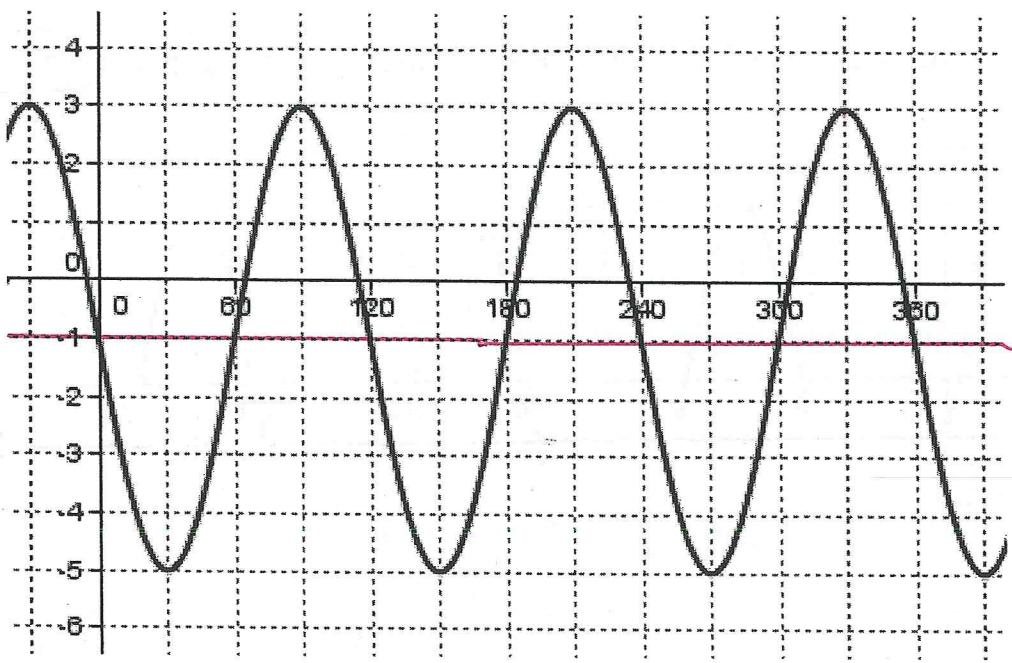
$$\therefore 120 = \frac{360}{k} \Rightarrow 120k = 360 \Rightarrow k = 3$$

$$* d = 60^\circ \text{ because } (0,0) \text{ moved to } (60^\circ, 0)$$

$$\therefore y = 4 \sin[3(x-60)] - 1$$

b) Find the equation of the graph using the base function $y = \cos x$ instead.

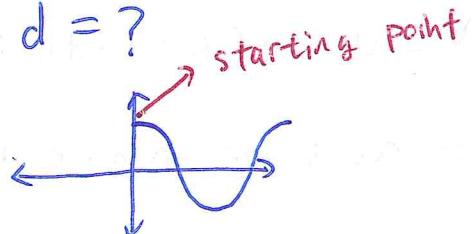
same
graph
as @



* $y = a \cos [k(x-d)] + c$

* $a = 4, c = -1, k = 3, d = ?$

* Since cosine start from crest,



d can be $90^\circ, 210^\circ, 330^\circ$ or even -30°

$$\therefore y = 4 \cos [3(x-90)] - 1 \quad \text{or}$$

$$y = 4 \cos [3(x+30)] - 1 \quad \text{or}$$

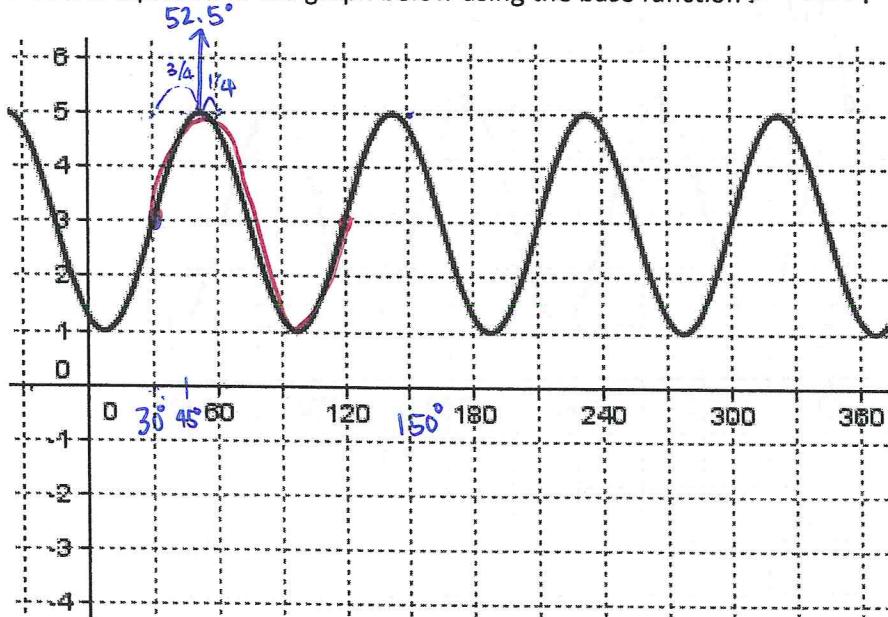
$$y = -4 \cos [3(x-30)] - 1$$

* Note: Sine curve and cosine curve have same a, c and k . Only d is different.

- Ex 2) a) $y = a \sin [k(x-d)] + c$
- * $c = \text{axis} = \frac{5+1}{2} = \frac{6}{2} = 3 \therefore c = 3$
 - * $a = \frac{(5-1)}{2} = 2 \therefore a = 2$
 - * Period = $150^\circ - 60^\circ = 90^\circ \rightarrow \frac{360}{k} = 90^\circ \rightarrow 90k = 360 \therefore k = 4$
 - * $d = 30^\circ$
- $$\therefore y = 2 \sin [4(x - 30^\circ)] + 3$$

Example 2

- a) Find the equation of the graph below using the base function $y = \sin x$.



$$\frac{3}{4} \times 30^\circ = \frac{90}{4} = 22.5$$

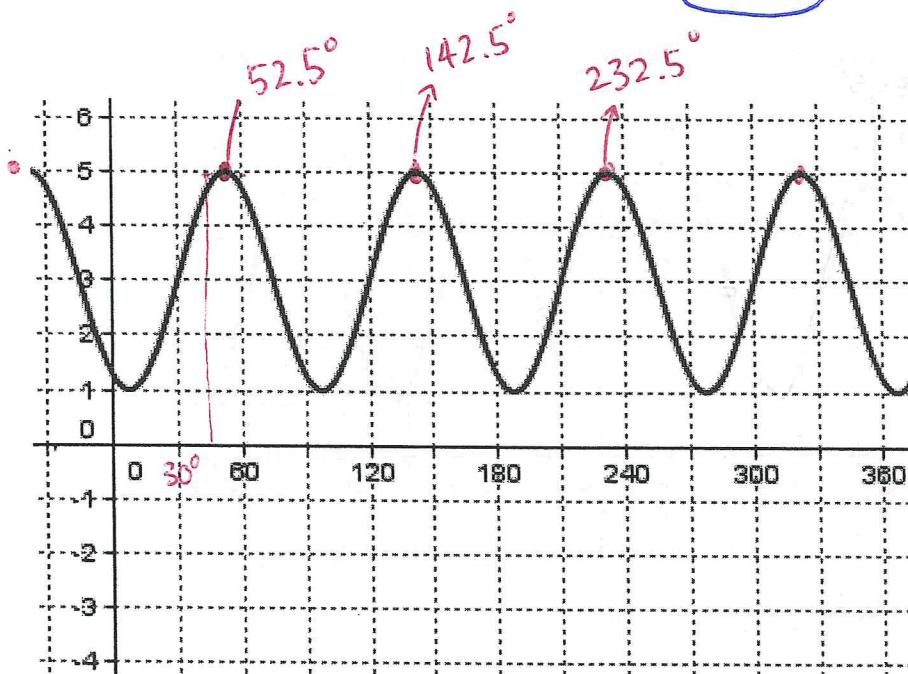
$$\therefore d = 30 + 22.5 = 52.5^\circ$$

$$* y = a \cos [k(x-d)] + c$$

$$* d = 52.5^\circ, 142.5^\circ, 232.5^\circ$$

$$\therefore y = 2 \cos [4(x - 52.5)] + 3$$

b) Find the equation of the graph using the base function $y = \cos x$ instead.

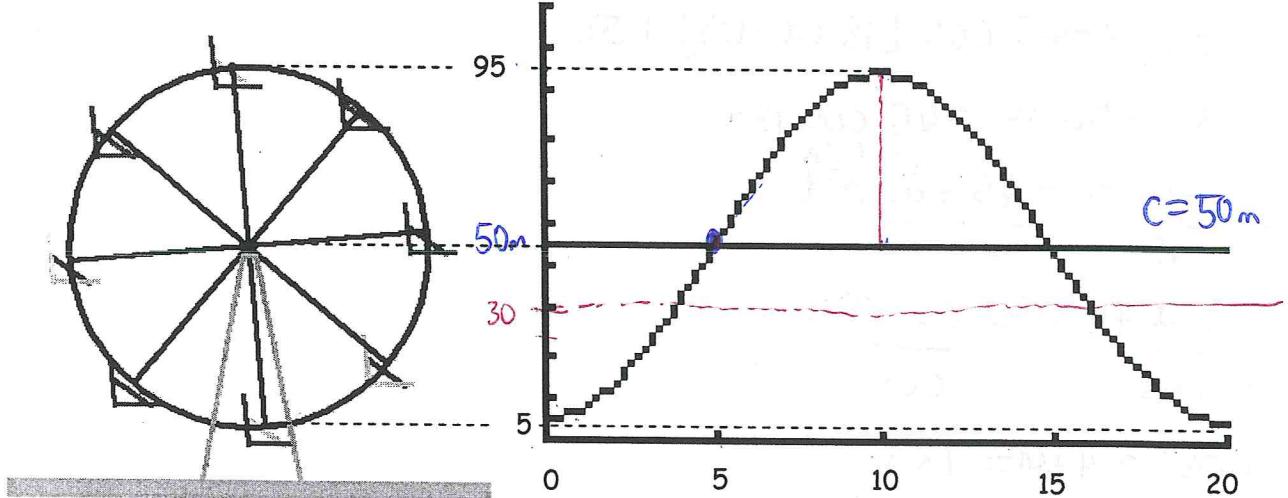


$$\frac{45+60}{2} = \frac{105}{2} = 52.5^\circ$$

Trigonometric Applications

There are many applications of trigonometric functions. Sound, light, tides, Ferris wheels, pendulums, and springs can all be modelled with sinusoidal graphs. In fact, anything that is periodic can be modelled by some combination of sine and cosine.

Example 1 A Ferris wheel has a radius of 45 m , and its centre is 50 m above the ground. The Ferris wheel makes a complete rotation in 20 seconds. Suppose the graph starts when the rider is at the lowest height.



a) Find an equation using the base function $y = \sin x \rightarrow y = a \sin [k(x-d)] + c$

$$* a = 45 = 95 - 50$$

$$* d = 5$$

$$* c = \frac{95+5}{2} = 50\text{ m}$$

$$\therefore y = 45 \sin [18(x-5)] + 50$$

$$* k \Rightarrow \text{period} \Rightarrow 20\text{m} = \frac{360}{k}$$

$$20k = 360$$

$$k = 18$$

b) Find an equation using the base function $y = \cos x$

$$y = 45 \cos [18(x-10)] + 50 \quad \text{or}$$

$$y = -45 \cos (18x) + 50$$

c) How high is the rider after 17 seconds?

$$y = 45 \cos [18(17-10)] + 50$$

$y = 23.5 \text{ m}$ ∴ The rider is at 23.5m after 17 seconds.

d) Between 0 and 20 seconds, when is the rider 30 m off the ground?

= restriction for x

$$30 = -45 \cos [18x] + 50$$

$$30 - 50 = -45 \cos 18x$$

$$\frac{-20}{-45} = \frac{-45 \cos 18x}{-45}$$

$$\frac{0.4444}{\cos} = \frac{\cos 18x}{\cos}$$

$$\cos^{-1} 0.4444 = 18x$$

$$\frac{63.612}{18} = \frac{18x}{18}$$

$$3.53 = x$$

↳ is the first point

e) Find an equation for the motion of a rider if the graph begins when the rider is at the highest point on the Ferris wheel.

When doing application questions, first draw a diagram of the situation. Then draw the graph using a sinusoidal function.