

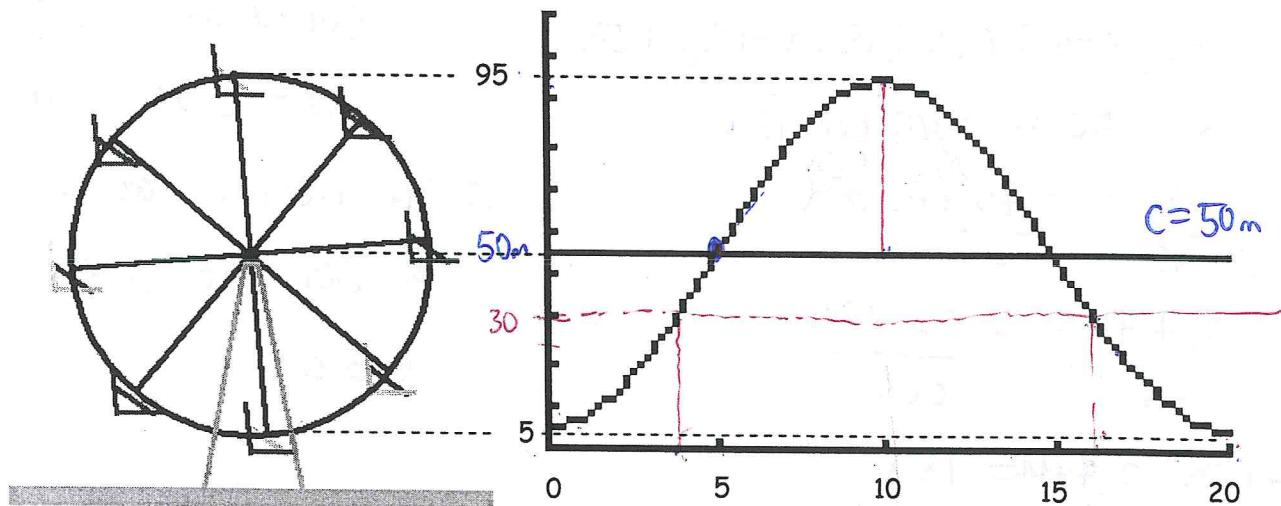
(May 25)

There are many applications of trigonometric functions. Sound, light, tides, Ferris wheels, pendulums, and springs can all be modelled with sinusoidal graphs. In fact, anything that is periodic can be modelled by some combination of sine and cosine.

Example 1

A Ferris wheel has a radius of 45 m , and its centre is 50 m above the ground.

The Ferris wheel makes a complete rotation in 20 seconds. Suppose the graph starts when the rider is at the lowest height.



a) Find an equation using the base function $y = \sin x \rightarrow y = a \sin [k(x-d)] + c$

$$\ast a = 45 = 95 - 50$$

$$\ast d = 5$$

$$\ast c = \frac{95+5}{2} = 50\text{m}$$

$$\therefore y = 45 \sin [18(x-5)] + 50$$

$$\ast k \Rightarrow \text{period} \Rightarrow 20\text{m} = \frac{360}{k}$$

$$\frac{20k}{20} = \frac{360}{20}$$

$$k = 18$$

b) Find an equation using the base function $y = \cos x$

$$y = 45 \cos [18(x-10)] + 50 \quad \text{or}$$

$$y = -45 \cos (18x) + 50$$

c) How high is the rider after 17 seconds?

$$y = 45 \cos [18(17-10)] + 50$$

$y = 23.5 \text{ m}$ ∴ The rider is at 23.5m after 17 seconds.

d) Between 0 and 20 seconds, when is the rider 30 m off the ground?

= restriction for x

$$30 = -45 \cos [18x] + 50$$

$$30 - 50 = -45 \cos 18x$$

$$\frac{-20}{-45} = \frac{-45 \cos 18x}{-45}$$

$$\frac{0.4444}{\cos} = \frac{\cos 18x}{\cos}$$

calculator $\cos^{-1} 0.4444 = 18x$

$$\frac{63.612}{18} = \frac{18x}{18}$$

$$3.53 = x$$

↳ is the first point

Second point is :

$$20 - 3.53 = 16.5 \text{ seconds}$$

∴ The rider is at a height
of 30m at 3.5s and
16.5 s.

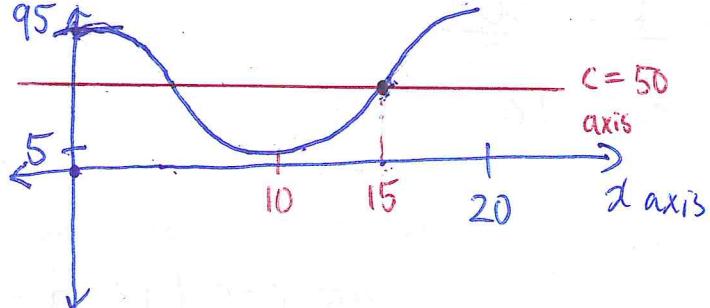
e) Find an equation for the motion of a rider if the graph begins when the rider is at the highest point on the Ferris wheel.

$$a = 45, c = 50, k = 18$$

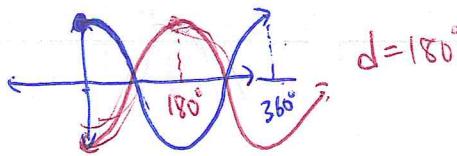
phase shift = none

$$\therefore y = 45 \cos(18x) + 50$$

$$\therefore y = 45 \sin[18(x-15)] + 50$$



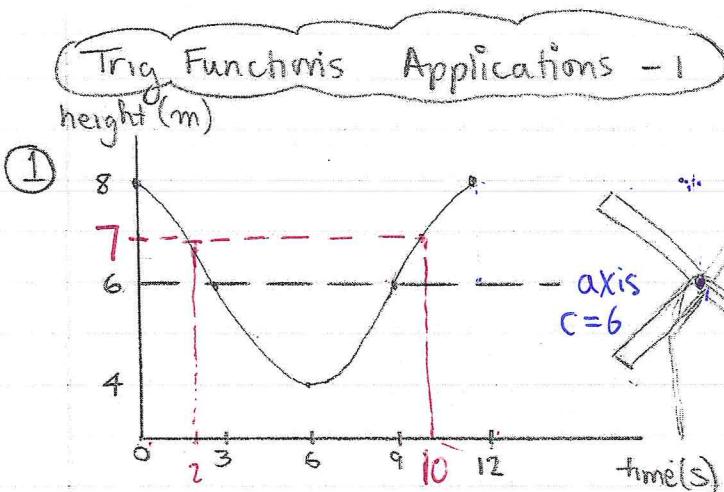
When doing application questions, first draw a diagram of the situation. Then draw the graph using a sinusoidal function.



"6m above the ground" = C

"blades 2m" = a.

"complete rotation in 12s" = period



$$\hookrightarrow 12 = \frac{360}{k}$$

$$\frac{12}{12} = \frac{360}{12}$$

$$k = 30$$

b) $a = \frac{y_{\max} - y_{\min}}{2}$

$$k = \frac{360}{12}$$

$$d = 0^\circ$$

$$c = \frac{y_{\max} + y_{\min}}{2}$$

$$= \frac{8 - 4}{2}$$

$$= 30$$

$$= \frac{8 + 4}{2}$$

= 2 ← length of blade

= 6 ← axis of symmetry

$$h = a \cos [k(t-d)] + c \quad \therefore H = 2 \cos [30t] + 6 \text{ or}$$

$$= 2 \cos [30t] + 6 \quad H = 2 \sin [30(t-9)] + 6$$

c) $h(5) = 2 \cos [30(5)] + 6$
 $= 4.3 \text{ m}$

$$h(40) = 2 \cos [30(40)] + 6$$

 $= 5 \text{ m}$

d) $\boxed{t=? \text{ when } H=7}$

$$7 = 2 \cos [30t] + 6 \quad \text{subtracted 6 from both sides}$$

$$1 = 2 \cos [30t] \quad \text{divide by 2 both sides}$$

$$\frac{1}{2} = \cos [30t]$$

CAST Rule... term arm is in Q1 or Q4

$$\cos^{-1} 0.5 = 30t$$

② $\rightarrow 30t = 60^\circ \text{ or } 30t = 360^\circ - 60^\circ \leftarrow \text{Q4}$

$$60 = 30t$$

$$t = 2 \text{ s}$$

$$30t = 300$$

$$2 = t$$

First answer

$$\uparrow$$

$$12 (\text{period}) - 2 = 10 \text{ seconds}$$

∴ When $t = 2$ seconds and 10 seconds, the Height is 7m.

t represent time in hours
Let S represent: surface temperature

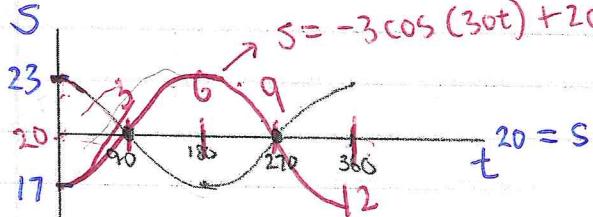
2) $S = -3 \cos(30t) + 20$ $t = ?$ when $S = 20^\circ\text{C}$

$$20 = -3 \cos(30t) + 20$$

$$0 = -3 \cos(30t)$$

$$0 = \cos(30t)$$

$$P = \frac{360}{30} = 12$$



$$30t = 90 \quad \text{or} \quad 30t = 270$$

$$t = 3$$

$$t = 9$$

t is the hours since 6:00 AM

$$6 + 3 = 9:00 \text{ Am}$$

$$6 + 9 = 15:00 \quad \text{or} \quad 3:00 \text{ PM}$$

