May 22

MPM2D
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$\longrightarrow$ Expanding
$\longleftarrow$ Factoring

Completing the Square

Recall: Expand.

1) $(x+5)^2$

$= (x+5)(x+5)$
$= x^2 + 5x + 5x + 25$
$= x^2 + 10x + 25$

2) $(x-4)^2$

$= (x-4)(x-4)$
$= x^2 - 4x - 4x + 16$
$= x^2 - 8x + 16$

3) $(x+p)^2$

$(a+b)^2 = a^2 + 2ab + b^2$
$(a-b)^2 = a^2 - 2ab + b^2$

These are all perfect squares.

How do we identify perfect squares trinomials? For example $x^2 - 16x + 64$

$a^2 \quad -2ab \quad b^2$

$x^2 - 16x + 64 = x^2 - 2(x)(8) + 8^2 = (x-8)^2$

$*a = x \quad * b = 8$

$a^2 - 2ab + b^2 \rightarrow (a-b)^2$

$(a-b)^2 = a^2 - 2ab + b^2$ <perfect squares>

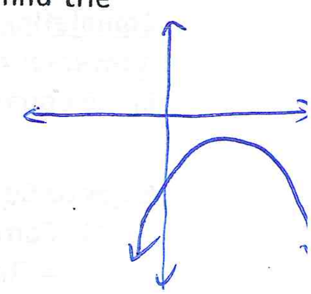
Why is this useful to us? Look at vertex form:

$$y = a(x-h)^2 + k$$

There is a perfect square in the equation, it is $(x-h)^2$

Why would we need to complete the square to find the vertex? Why can't we just find the midpoint of the zeros and plug it in?

Sometimes, the quadratics does not have any zeros.



Investigating Perfect Squares:

Find the value of m that makes the expression a perfect square, then write out the expression as a perfect square:

Example 1

a) $x^2 + 4x + m \; ?$

$a^2 + 2ab + b^2$

$a = x \qquad b^2 = m$

$\frac{2xb}{2x} = \frac{4x}{2x}$

$b = 2$

$\therefore (x+2)^2$

$\therefore m = 4$

b) $x^2 - 4x + m$

$a^2 - 2ab + b^2$

$a^2 = x^2 \rightarrow a = x$

$b^2 = m$

$\frac{-2xb}{-2x} = \frac{-4x}{-2x}$

$b = 2$

$\therefore m = 4$

$\therefore (x-2)^2$

c) $x^2 - 6x + m$

$a^2 - 2ab + b^2 = (a-b)^2$

$x = a$, $m = b^2$, $-6x = -2ab$

$\frac{-6x}{-2x} = \frac{-2xb}{-2x}$ $\therefore (x-3)^2$

$3 = b$ $\therefore m = 9$

d) $x^2 - 10x + m$

$a^2 - 2ab + b^2 = (a-b)^2$

$x = a$, $m = b^2$, $-2ab = -10x$

$\frac{-2xb}{-2x} = \frac{-10x}{-2x}$ $\therefore (x-5)^2$

$b = 5$ $\therefore m = 25$

Investigate changing vertex form into standard form:

Example 2 Find standard form of the equation $y = 2(x-3)^2 + 4$

Standard form $y = ax^2 + bx + c$

$a = x$ $b = 3$

$a^2 + b^2 - 2ab$

$y = 2(x-3)(x-3) + 4$

$y = 2(x^2 + 3^2 - 2(x)(3)) + 4$

$y = 2(x^2 + 9 - 6x) + 4$

$y = 2x^2 + 18 - 12x + 4$

$y = 2x^2 - 12x + 22$

$y = ax^2 + bx + c \rightleftarrows y = a(x-h)^2 + k$

Completing the Square changes standard form into vertex form:

It reverses the steps of changing vertex form into standard form, by ***creating a perfect square*** in the equation.

$(a-b)^2$

$(a+b)^2$

Steps to Completing the Square:

1) Common factor the number in front of the x out of both the x^2 and x-term
   = Take out a from ax^2 by using common factoring
2) Find the constant that must be added and subtracted to create a perfect square.
   This is done by:
   i) Using the **coefficient of the x-term** from step 1
   ii) Dividing that coefficient by 2
   iii) Squaring that coefficient.
3) Group the three terms that form the perfect square. Move the subtracted value outside the brackets by multiplying it by the common constant factor.
4) Factor the perfect square and collect like terms.

Change the following standard form into vertex form.

Example 3 ~~Find the standard form of each equation.~~

a) $y = x^2 - 8x \rightarrow (x-h)^2 + k$

$y = x^2 - 8x + \left(\frac{-8}{2}\right)^2 - \left(\frac{-8}{2}\right)^2$

$y = x^2 - 8x + 16 - 16$

$a^2 \quad -2ab + b^2 \qquad a = x, \; b = 4$

$y = (x-4)^2 - 16$

$\therefore$ Vertex $= (4, -16)$

b) $f(x) = 2x^2 + 12x$

c) $g(x) = 2x^2 + 12x - 3$

d) $y = 5x^2 + 10x - 11$

e) $h(x) = -3x^2 + 6x - 7$

f) $y = -2x^2 - 4x + 5$

g) $q(x) = \frac{1}{2}x^2 - 4x + 9$

h) gravity question :

$h(t) = -4.9t^2 + 10.78t + 1.6$

Homework: Worksheet