Change the following standard form into vertex form.

Example 3 ~~Find the standard form of each equation.~~

May 25 (Mon)

* Test on Friday (May 29)

a) $y = x^2 - 8x \longrightarrow (x-h)^2 + k$

$y = x^2 - 8x + \left(\frac{-8}{2}\right)^2 - \left(\frac{-8}{2}\right)^2$

$y = x^2 - 8x + 16 - 16$

$a^2 \quad -2ab + b^2 \quad \left[\begin{array}{l} a = x \\ b = 4 \end{array}\right.$

$y = (x-4)^2 - 16$

$\therefore$ Vertex $= (4, -16)$

b) $f(x) = 2x^2 + 12x$

$= 2(x^2 + 6x)$

$= 2\left[x^2 + 6x + \left(\frac{6}{2}\right)^2 - \left(\frac{6}{2}\right)^2\right]$

$a^2 + 2ab + b^2 \rightarrow b = 3,\ a = x$

$= 2\left[(x+3)^2 - 9\right]$

$= 2(x+3)^2 + (2 \times -9)$

$\therefore f(x) = 2(x+3)^2 - 18$

c) $g(x) = 2x^2 + 12x - 3$

$g(x) = 2(x^2 + 6x) - 3$

$= 2\left[x^2 + 6x + \left(\frac{6}{2}\right)^2 - \left(\frac{6}{2}\right)^2\right] - 3$

$= 2(x^2 + 6x + 9 - 9) - 3$

$a^2 + 2ab + b^2 \quad \left[\begin{array}{l} a = x \\ b = 3 \end{array}\right.$

$= 2\left[(x+3)^2 - 9\right] - 3$

$= 2\left[(x+3)^2\right] - 3 + (-9 \times 2)$

$g(x) = 2(x+3)^2 - 21$

d) $y = 5x^2 + 10x - 11$

$y = 5(x^2 + 2x) - 11$

$y = 5\left[x^2 + 2x + \left(\frac{2}{2}\right)^2 - \left(\frac{2}{2}\right)^2\right] - 11$

$a^2 + 2ab + b^2 \rightarrow (a+b)^2$

$y = 5\left[(x+1)^2 - 1\right] - 11$

$y = 5(x+1)^2 - 11 + (5 \times -1)$

$\therefore y = 5(x+1)^2 - 16$

e) $h(x) = -3x^2 + 6x - 7$

$= -3(x^2 - 2x) - 7$

$= -3\left[x^2 - 2x + \left(\frac{-2}{2}\right)^2 - \left(\frac{-2}{2}\right)^2\right] - 7$

$a = x \quad b = -1$

$= -3\left[(x-1)^2 - 1\right] - 7$

$= -3(x-1)^2 - 7 + (-3 \times -1)$

$= -3(x-1)^2 - 4$

$\therefore g(x) = -3(x-1)^2 - 4$

f) $y = -2x^2 - 4x + 5$

$y = -2(x^2 + 2x) + 5$

$y = -2\left(x^2 + 2x + \left(\frac{2}{2}\right)^2 - 1^2\right) + 5$

$a = x \quad b = 1$

$y = -2\left[(x+1)^2 - 1\right] + 5$

$y = -2(x+1)^2 + 5 + (-2 \times -1)$

$\therefore y = -2(x+1)^2 + 7$

g) $q(x) = \frac{1}{2}x^2 - 4x + 9$

$$g(x) = \frac{1}{2}(x^2 - 8x) + 9$$
$$= \frac{1}{2}\left(x^2 - 8x + \left(\frac{-8}{2}\right)^2 - (-4)^2\right) + 9$$
$a = x \quad b = -4$
$$= \frac{1}{2}\left[(x-4)^2 - 16\right] + 9$$
$$= \frac{1}{2}(x-4)^2 + 9 + \left(\frac{1}{2} \times -16\right)$$
$$= \frac{1}{2}(x-4)^2 + 9 - 8$$
$$\therefore g(x) = \frac{1}{2}(x-4)^2 + 1$$

h) gravity question :

$h(t) = -4.9t^2 + 10.78t + 1.6$

$$h(t) = -4.9(t^2 - 2.2t) + 1.6$$
$$= -4.9\left(t^2 - 2.2t + \left(\frac{-2.2}{2}\right)^2 - (-1$$
$t = a \quad -1.1 = b$
$$+$$
$$= -4.9(t-1.1)^2 + 1.6 + (-1.21 \times -4$$
$$= -4.9(t-1.1)^2 + 1.6 + 5.929$$
$$= -4.9(t-1.1)^2 + 7.529$$
$$\therefore h(t) = -4.9(t-1.1)^2 + 7.529$$

Homework: Worksheet

May 25 HW solution MPM2D Park

1. d) $y = x^2 - 12x + 2$

$y = x^2 - 12x + \left(\frac{-12}{2}\right)^2 - \left(\frac{-12}{2}\right)^2 + 2$

$y = (x-6)^2 - \frac{144}{4} + 2$

$y = (x-6)^2 - 36 + 2$

$\therefore y = (x-6)^2 - 34$

e) $y = x^2 + 14x + \left(\frac{14}{2}\right)^2 - \left(\frac{14}{2}\right)^2 + 39$

$y = (x+7)^2 - 49 + 39$

$y = (x+7)^2 - 10$

2. b) $y = x^2 - 24x + 215$

$y = x^2 - 24x + \left(\frac{-24}{2}\right)^2 - \left(\frac{-24}{2}\right)^2 + 215$

$y = (x-12)^2 - 144 + 215$

$y = (x-12)^2 + 71$ $\therefore$ Vertex $= (12, 71)$

4.e) $y = -3x^2 + 18x - 25$

$y = -3(x^2 - 6x) - 25$

$y = -3\left(x^2 - 6x + \left(\frac{-6}{2}\right)^2 - \left(\frac{-6}{2}\right)^2\right) - 25$

$y = -3(x-3)^2 - 25 + (-3 \times -9)$

$y = -3(x-3)^2 + 2$ $\therefore$ Vertex $= (3, 2)$

axis of symetry $\Rightarrow$ $x = 3$

when $x = 0$, $\rightarrow y = -25 \rightarrow (0, -25)$

when $x = 1$, $\rightarrow y = -3 + 18 - 25 = -10 \rightarrow (1, -10)$

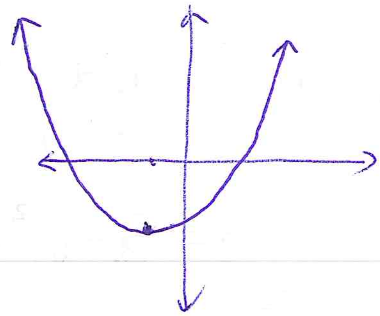
5. a) $y = 1.5x^2 + 6x - 8$

$y = 1.5(x^2 + 4x) - 8$

$y = 1.5[x^2 + 4x + (\frac{4}{2})^2 - (\frac{4}{2})^2] - 8$

$y = 1.5(x+2)^2 - 8 + (-4 \times 1.5)$

$y = 1.5(x+2)^2 - 14$ $\therefore$ Vertex $= (-2, -14)$



It opens up because "a" is positive. So vertex is minimum point. Minimum $= -14$

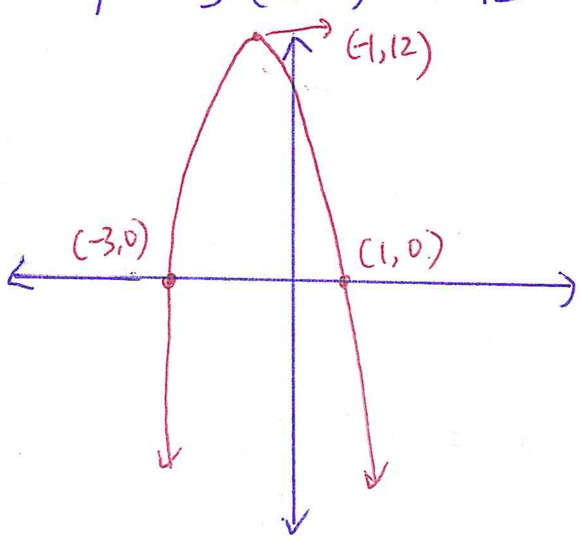
6. c) x intercepts $= (1, 0)$ and $(-3, 0)$

$y = -3(x^2 - 3 + 2x)$

$y = -3(x^2 + 2x + (\frac{2}{2})^2 - (\frac{2}{2})^2 - 3)$

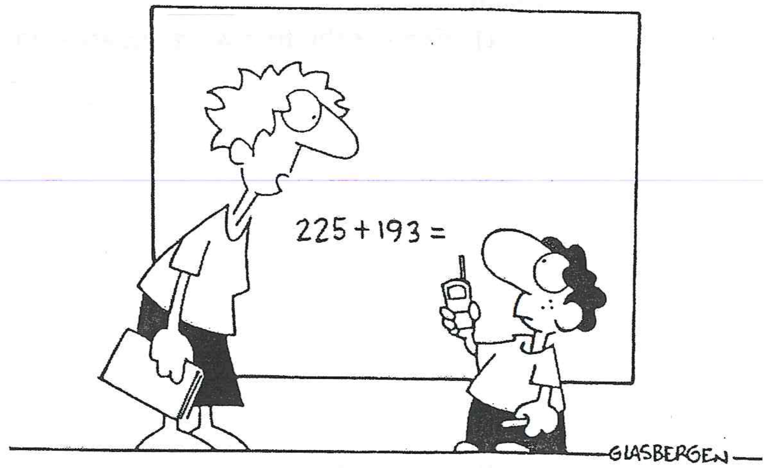
$y = -3(x+1)^2 + (-4 \times -3)$

$y = -3(x+1)^2 + 12$ $\therefore$ Vertex $= (-1, 12)$



Completing the Square WORKSHEET

1. Write each quadratic function in the form $y = a(x - h)^2 + k$.
   a) $y = x^2 + 6x$
   b) $y = x^2 + 8x + 3$
   c) $y = x^2 - 4x - 5$
   d) $y = x^2 - 12x + 2$
   e) $y = x^2 + 14x + 39$



"You have to solve this problem by yourself. You can't call tech support."

2. Identify the vertex of each function by completing the square.
   a) $y = x^2 + 2x + 7$
   b) $y = x^2 - 24x + 215$
   c) $y = x^2 + 8x$
   d) $y = x^2 - 6x + 9$
   e) $y = 14 - 16x + x^2$

3. Determine the following for each quadratic function shown below: the direction of opening, the coordinates of the vertex, the equation of the axis of symmetry, the domain and range, and the maximum/minimum value and when it occurs.
   a) $y = -x^2 + 10x + 7$
   b) $y = 2x^2 + 12x + 65$
   c) $y = -3x^2 + 12x - 17$
   d) $y = 4x^2 + 16$
   e) $y = -7x^2 + 14x + 3$
   f) $y = -0.5x^2 + 4x - 5$
   g) $y = 5x^2 - 30x$

4. Sketch the graph of each function. Show the coordinates of the vertex, the equation of the axis of symmetry, and the coordinates of two other points on the curve.
   a) $y = x^2 - 2x - 8$
   b) $y = x^2 + 4x$
   c) $y = -x^2 - 6x - 9$
   d) $y = \frac{1}{2}x^2 + 2x + 1$
   e) $y = -3x^2 + 18x - 25$

5. Without graphing each function, state whether it has a maximum or minimum value. Determine the maximum or minimum value and when it occurs.
   a) $y = 1.5x^2 + 6x - 8$
   b) $y = 20x - 0.2x^2$
   c) $y = 2x + 1 + 0.1x^2$
   d) $y = -0.003x^2 + 0.6x - 11$

6. Sketch the graph of each function and state the coordinates of the vertex.
   a) $y = (x + 1)(x - 3)$
   b) $y = (2x + 1)(x - 2)$
   c) $y = -3(x - 1)(x + 3)$

Hint: Expand first

MPM2D

Ms. Kueh

Completing the Square - Applications

Recall:

1) Express the following equation in vertex form.

$$y = 2x^2 + 20x + 43$$

2) A ball is thrown in the air. It's height, in metres, after t seconds is $h(t) = -5(t-6)^2 + 40$. What was the maximum height of the ball? When did it reach the maximum height?

Maximum/Minimum Questions

A quadratic relation reaches a maximum/minimum at __________________.

In problems where they ask for a ________________________________ you will be required to find _______________. To do this you want to take the equation given and put it into _________________ form by ____.

Example 1 A baseball player hits a baseball into the air. The motion of the ball is modeled by the equation $h(t) = -5t^2 + 20t + 1$.

*note that -5 is rounded from -4.905, which you would use in a physics class.

a) What was the height of the ball when it was hit?