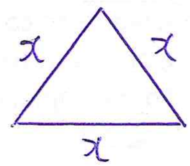
May 26 Park * Unit Test will occur on Friday June 5.

MCR3U **Intro to Sequences**



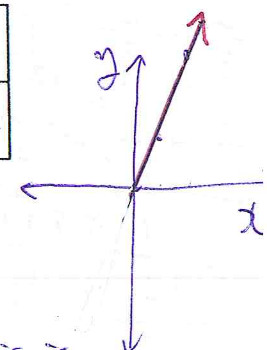
$P = x + x + x$

$P = 3x$

Investigate the following situations.

1. Fill in the chart below for an equilateral triangle.

Side Length	1cm	2cm	3cm	4cm	5cm
Perimeter	3 cm	6cm	9cm	12cm	15cm



a) Predict the perimeter of the triangle if the side length is . 20 cm

$20 \times 3 = 60$

b) Identify the type of relationship between the perimeter and the side length.
Hint: Examine the first differences.

$15 - 12 = 3$ $12 - 9 = 3$ $9 - 6 = 3$ etc. FD are constant, so this is linear relations

c) Create a formula that can be used to find the perimeter of an equilateral triangle with side length .

$y = 3x$ or $P = 3n$

$P, y =$ Perimeter
$x, n =$ Side length

d) Verify that your formula works.

When $n = 5 \rightarrow P = 3(5) = 15$ ✓

2. The cells in a culture divide every hour.

a) If there are 500 cells now, how many cells will there be in 1 hour?

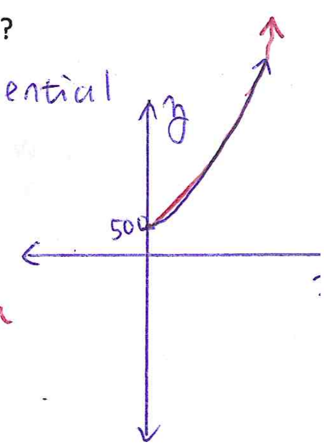
$500 \times 2 = 1000$ cells

b) Create a table showing the number of cells at the end of each hour for hours.

Time (hours)	1	2	3	4	5	x
Cells	1000	2000	4000	8000	16000	y

c) What type of relationship exists between the number of cells and the time?

$\frac{2000}{1000} = 2$ $\frac{4000}{2000} = 2$ ∴ Relationship is exponential



d) Predict the number of cells after 8 hours

$500 \times 2^8 = 128\,000$

e) Create an equation that relates the number of cells to the time.

$B = 500 \times 2^n$

$B =$ total # of bacteria
$n =$ time in

f) Verify that your formula works.

Check when $= 5$ hours

$B = 500 \times 2^5 = 16000$ bacteria

Unit: 2 — [1 | 1], [2]

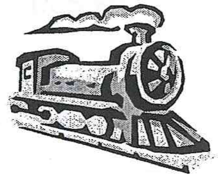
Unit 3 — [1 | 2], [2 | 1], [1 | 1 | 1]

Unit 4 — [1 | 1 | 1 | 1], [2 | 2], [1 | 2 | 1], [1 | 1 | 2], [2 | 1 | 1]

Unit 5 — [1 | 1 | 1 | 1 | 1], [1 | 1 | 1 | 1 | 1] etc

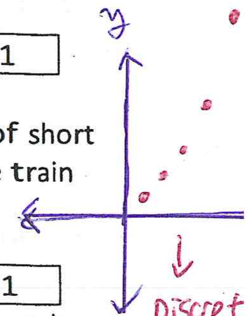
Unit (x)	# possibility (y)
1	1
2	2
3	3
4	5 = 3+2 (add two previous y values)
5	8 = 5+3
6	13 = 8+5
7	21
8	34
9	55
10	89
11	144
12	233

3. Elizabeth wants to construct a train of total length 12 units using cars which are unit long or 2 units long. The question is, how many different trains are there? For example, here is one possibility, using 6 cars of length 1 and 3 cars of length 2.



1	1	1	2	2	1	2	1	1

Other possibilities are obtained by rearranging the cars, or using different numbers of short and long cars. Note that train has a front and a back, so that the mirror image of the train diagrammed above is counted as a different train.

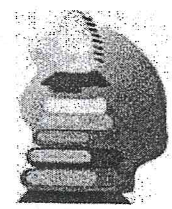


1	1	2	1	2	2	1	1	1

(Hint: Try a simpler problem first, eg. # of possibilities for a 1 car train, a 2 car train. etc)

∴ There are 233 possibilities for a train (of length 12)

4. The Tower of Hanoi puzzle involves moving a tower of disks from one of three pegs to another peg in the least number of moves. The disks have different sizes and are arranged on the first peg from largest to smallest, with the largest on the bottom and the smallest on top. Disks are moved according to the following set of rules:



i. Only one disk can be moved at a time.
ii. Only the top disk can be moved.
iii. Never place a larger disk on top of a smaller disk.

a) What is the least number of moves required to move five disks?
(Hint: Start with 1 disk, then 2, etc.)

b) Predict how many moves would be required if there were blocks on the first peg.

c) What is the general relationship between the number of blocks and the number of moves required?

5. In each of these examples a pattern can be found. Try to find the pattern and determine the next two terms for each of the following.

a) 1, 3, 5, 7, 9, 11 (+2)

b) 12, 24, 48, 96, 192 (×2)

c) $\frac{3}{4}, \frac{5}{9}, \frac{7}{16}, \frac{9}{25}, \frac{11}{36}$ (numerator: increase by 2 denominator: FD increase by 2) $(n+1)^2$

These are examples of sequences of numbers. A ***sequence*** is a set of numbers, shapes, letters, etc. that are in a distinct or recognizable pattern. Each number in a sequence is called a ***term.*** * For sequence, domain is always natural number. → graphs are just points (not a curve or line)

6. In the sequence 3, 6, 9, 12, 15, 18 ∴ the fourth term is 12. We write this as $t_4 = 12$. State the values of t_2 and t_6. The terms of this sequence represent the perimeters found in question 1 above. We found that $P = 3n$ where n represents the side length. For the sequence . Find and .

$t_2 = 6$, $t_6 = 18$

$t_{30} = ? = 3(30) = 90$

$t_{50} = ? = 3(50) = 150$

7. Given the general term, state the first terms of each sequence:

a) $t_n = 3n + 1$

$t_1 = 3(1) + 1 = 4$

$t_2 = 3(2) + 1 = 7$

$t_3 = 3(3) + 1 = 10$

∴ sequence is 4, 7, 10 etc...

b) $t_n = 2^n$

$t_1 = 2^1 = 2$

$t_2 = 2^2 = 4$

$t_3 = 2^3 = 8$

$t_4 = 2^4 = 16$

∴ Sequence is 2, 4, 8, 16

Follow-up Questions

1. You have 7 steps to climb. You can go up 1 step or 2 steps at a time. In how many different ways can you climb the steps. Use the chart below to help organize your work.

Number of steps	1	2	3	4	5	6	7
Number of possible ways	1	2	3	5	8	13	21

2. Using nickels and dimes only, in how many different ways can you make up various sums of money? (i.e. \$0.05, \$0.10, \$0.15, \$0.20, \$0.25, \$0.30) etc
(i.e.,); etc.

1 2 2 3 3 4

\$0.50 = ? How many possibilities?

Answer is 6 possibilities.

In Sequence **Homework:** Pg. 360 C1, C2, (1-6)first and last, 8, 11-13 Thinking #18

* Sometimes, you may use formula, but other times, you can't find a formula. You just have to find a pattern.

* Sequence is called a discrete function because it is set of points.