May ~~26~~ 27 Park Unit 7 Day 2

MCR3U

Recursion Formulas

The value of a term is based on the value of the previous term(s).

Eg. Fibonacci 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, ...

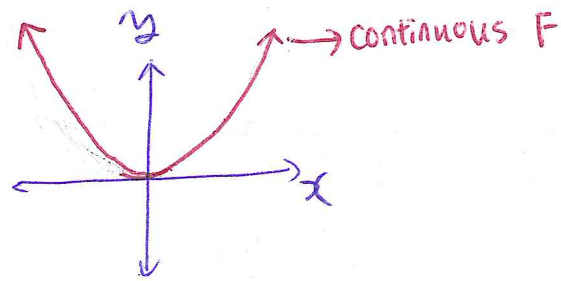
To find the value of one term, add the previous two terms together

$t_5 = t_3 + t_4$ $t_5 = t_3 + t_4$ or $f(5) = f(3) + f(4)$

$t_1 = 1,$ $t_2 = 1,$ $t_n = t_{n-2} + t_{n-1}$ → one term before t_n

Alternatively, you could write this in function notation:

$f(n) = f(n-2) + f(n-1)$



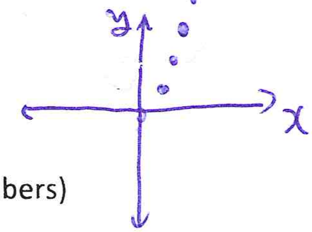
Continuous and Discrete Functions

Continuous Functions – domain is $x \in R$

Example: $y = x^2$

Discrete Functions – domain is $n \in N$ (natural number such as 1, 2 3, 4, 5

Example: $f(n) = n^2$



* Sequence is discrete function.

Example 1 Write the first 4 terms of each sequence, $n \in N$ (natural numbers)

a) $f(1) = 5$, $f(n) = f(n-1) - 4$

$f(2) = f(2-1) - 4$
$= f(1) - 4$
$= 5 - 4 = 1$

$\therefore f(2) = 1$

$f(3) = f(3-1) - 4$
$= f(2) - 4$
$= 1 - 4 = -3$

$\therefore f(3) = -3$

$f(4) = f(4-1) - 4$
$= f(3) - 4$
$= -3 - 4 = -7$

$\therefore$ Sequence = 5, 1, −3, −7 ....

b) $t_1 = 3$, $t_n = 2t_{n-1} - n$

$t_2 = 2t_{2-1} - 2$
$t_2 = 2t_1 - 2$
$= 2 \cdot 3 - 2$
$= 6 - 2 = 4$

$t_3 = 2t_{3-1} - 3$
$= 2t_2 - 3$
$= 2 \cdot 4 - 3$
$= 8 - 3 = 5$

$t_4 = 2t_{4-1} - 4$
$= 2 \cdot t_3 - 4$
$= 2 \cdot 5 - 4$
$= 10 - 4 = 6$

$\therefore$ Sequence = 3, 4, 5, 6 ....

Example 2

Determine a recursion formula for

n: 1 2 3 4

a) $-2, 7, 16, 25, \ldots$

+9 +9 +9

last term

first term

Use t_n notation: $t_n = t_{n-1} + 9$, $t_1 = -2$

Use $f(n)$ notation: $f(n) = f(n-1) + 9$, $f(1) = -2$

t_1 t_2 t_3

b) $1, -3, 9, -27$

×−3 ×−3 ×−3

last term

Use t_n notation: $t_n = -3t_{n-1}$, $t_1 = 1$

Use $f(n)$ notation: $f(n) = -3f(n-1)$, $f(1) = 1$

e.g. $t_1 = 1$

Don't forget you need a starting value when you define your recursion formula!

Example 3

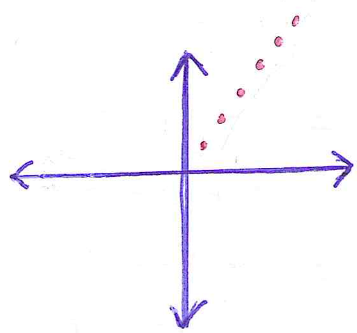
formula, which depends on the previous term

Find the recursive formula for: 2, 4, 6, 8, 10, 12, 14, 16

$t_n = t_{n-1} + 2$ or $f(n) = f(n-1) + 2$

Find an explicit formula for the same sequence (a formula that does *not* depend on the previous terms). **Hint:** Can you graph the sequence in some way?

x (n)	y (t_n)
1	2
2	4
3	6
4	8
5	10



$\therefore$ Explicit Formula: $t_n = 2n$

or $f(x) = 2x$

or $y = 2x$